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SCHOOL OF ENGINEERING

CONSOLIDATED SEMIANNUAL PROGRESS REPORT

NO. 3

Covering Research Activity During the Period

1 October 1965 through 31 March 1966

ECTRONIC SCIENCES LABORATORY

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Prepared by
Electronic Sciences Laboratory
of the
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I. SOLID STATE

1.1 SEMICONDUCTORS

A. Electron Probe Analysis of Semiconductors

Grant AF-AFOSR-76-66, Air Force Office of Scientific Research

D. B. Wittry, D. F. Kyser, E. L. Miller, T. Rao Sahib,

J. McCoy, and A. Van Couvering

The research performed since the last semiannual report has concentrated on the influence of surface recombination on cathodoluminescence in GaAs but investigations have also continued on the influence of temperature, uniaxial strain, and impurities on the intensity of cathodoluminescence. A brief description of the status and results of these investigations follows:

a) The Voltage Dependence of Cathodoluminescence

When the accelerating voltage of an electron beam is increased at constant electron beam power, the intensity of cathodoluminescence from GaAs increases. These results can be explained if non-radiative surface recombination occurs and if the diffusion length of the excess carriers is larger than the penetration depth of the electron beam at low voltages. Using information on the distribution of excitation with depth in the specimen obtained from x-ray absorption correlation functions[1], it is possible to predict the variation of intensity for the long wave length portion of the spectrum with beam voltage (i.e. the region in which internal absorption is negligible). The result, assuming that intensity is directly proportional to the net excess carrier concentration, is:

$$I \sim \varepsilon \left[1 - \frac{S}{S+1} \frac{1}{(1+w)} \frac{1}{1 + \left(\frac{h}{h+1}\right) w} \right] \qquad (1)$$

where ε is the bulk radiative recombination efficiency, S the reduced surface recombination velocity (surface recombination velocity/diffusion velocity), $h = 1.2A/Z^2$ with A = atomic weight and Z = atomic number. w is given by the following:

$$w = \frac{V^n}{1820(30^n - 9^n)\rho L} \qquad (2)$$

where V = the accelerating voltage (kv), L = the diffusion length and ρ the density. The variation of w with voltage is due to the variation in electron range with electron beam energy; it is assumed that the exponent n is constant over the range of 5 kv to 50 kv.

Experimental results obtained cannot be with an exponent of 1.5 or 1.7 as expected from existing knowledge of the range energy relations. The discrepancies are attributed to (i) a non-linear relationship between carrier generation and the intensity of recombination radiation observed on some specimens[2], or to (ii) the existence of a dead layer at the surface. A more detailed theory which takes account of the existence of a dead layer gives the following result:

$$I \sim \left[\frac{1 + \mathrm{Erf}\, a}{1 + \mathrm{Erf}\,(u_o/\Delta u)}\right] - \left[\frac{S}{S+1} \quad \frac{(1 + \mathrm{Erf}\, b)\ \mathrm{Exp}(b^2 - a^2)}{1 + \mathrm{Erf}\,(u_o/\Delta u)}\right] \tag{3}$$

where

$$\mathrm{Erf}\, a = \frac{2}{\sqrt{\pi}} \int_o^a e^{-t^2}\, dt$$

$$a = \left(u_o - \frac{d}{w'L}\right)\frac{1}{\Delta u}$$

$$a = \left(u_o - \frac{d}{w'L} - \frac{\Delta u^2 w'}{2}\right)\frac{1}{\Delta u}$$

d is the thickness of the dead layer and u_o and Δu are constants in the assumed distribution of excitation with depth, namely:

$$\phi(u)\, du \sim \exp -\left(\frac{u - u_o}{\Delta u}\right)^2 du \tag{4}$$

where $u = \rho z/R$ and R is the maximum range in gm/cm^2. Values of u_o = 0.125 and Δu = 0.350 were obtained by D. Brown from the energy loss in a Cu specimen at 30 kv using the transport model described in another section of this report (see page 9). Families of curves given by Eq. 3 were plotted with the aid of a computer and have been used to interpret the experimental data with the following assumptions: (1) the diffusion length and surface recombination velocity are constant, (2) the number of excess carriers produced is always small compared to the number of majority carriers present at equilibrium, (3) the solid angle received by the detector is independent of the source depth, (4) the effective number of carriers is directly proportional to the power in the incident beam, and (5) the reflection coefficient does not depend on the source depth.

In specimens for which these assumptions are valid, good agreement between theory and experimental values of diffusion length can be obtained in the range of 0.4 to 3 microns with an estimated accuracy of +30%. This work is now being prepared for publication and further experiments are planned to aid in the understanding of those cases for which theory and experiment do not agree.

b) Temperature Dependence of Cathodoluminescence

Attempts were made to determine spectral characteristics of cathodoluminescence in gallium arsenide as a function of carrier concentration over the range of 100-300°K, using the temperature-controlled stage developed at this laboratory[3]. Room temperature results were in agreement with those reported previously[4], but low temperature results were inconsistent due to the rapid condensation of oil and water vapor on the sample surface. Low temperature measurements of diffusion length were also attempted using the method described in (a) above but the condensation of vapors resulted in a gradual decrease of signal. Modifications of the electron microprobe are now underway to reduce the concentration of vapors in the vacuum system using freon and liquid-nitrogen cooled surfaces.

c) Strain Effects in Cathodoluminescence

Research is continuing on the effect of strain on the intensity and polarization of the cathodoluminescence from GaAs. A beam voltage of

50 kv is now being used to increase the depth of penetration of the electron beam, thus minimizing possible relief of stress by the free surface. Better signal strength is also achieved with the higher voltage. Measurements have been complicated by the fact that partial horizontal polarization is introduced by the optical viewing system. A method has been developed which takes into account this partial polarization, and which also eliminates the need to maintain exactly identical conditions of measurement before and after application of the stress.

d) Impurity Variations in Te-Doped GaAs

Measurements of the tellurium concentration by electron probe microanalysis have been correlated with the local variations in cathodoluminescence efficiency. Earlier attempts[5] to observe such a correlation failed because the practical limits of detectability in the electron probe microanalyzer are of the same order as the solubility limit of Te in GaAs.

The detection limit in microanalysis is established by the x-ray line/background ratio and the measurement time. If the line/background ratio is fixed, the limit of detectability can be improved only by increasing the time of measurement; an upper limit is then imposed by contamination of the specimen and by instrument stability. In most cases, the problem of observing concentration fluctuations is further worsened by the need for a series of measurements at many points on the specimen under identical conditions of analysis.

In the present work, a 400 channel digital signal-averaging device was used to obtain the average x-ray counting rate as a function of position of the electron probe on the specimen. The probe was swept by electrostatic deflection perpendicular to the focal circle of the x-ray spectrometer using the amplified sweep from the signal averager as input to the sweep driver in the scanning system of the microanalyzer[6]. The electron beam was run at 30 kev and 0.5 μa with a sweep of 300 microns in 16 seconds. The time constant in the electronic integrator was selected as 0.1 sec. to provide smoothing without sacrifice of spatial resolution. Similar results could also be obtained using a multiscaling mode and taking running averages to provide smoothing. For the Te $L_{\alpha 1}$ line recorded with a LiF crystal in a 4" radius spectrometer the line/background ratio was 495 (referred to 100% Te in GaAs). The average counting rate on the specimen with a carrier concentration of $5.2 \cdot 10^{18}/cm^3$ was 237 counts/sec.

The x-ray signal averaged over 400 scans is compared with the instantaneous IR signal in Fig. 1. Trace A, made with the x-ray and IR signals of opposite polarity, shows that the IR intensity increases as the x-ray counting rate decreases. The result is in agreement with measurements on specimens with various carrier concentrations by Cusano[7] and Casey[4]. Similar correlations were observed on another specimen of GaAs from the same ingot using a 1024 channel signal averager, 160 sweeps of 128 sec. duration with a 1 second time constant. The zero of the x-ray signal is 22 divisions above the average value. The average Te concentrations, obtained by applying corrections[8] for absorption and atomic number assuming α is 1.15 is 0.030 wt% or 170 ppm (atomic). The concentration

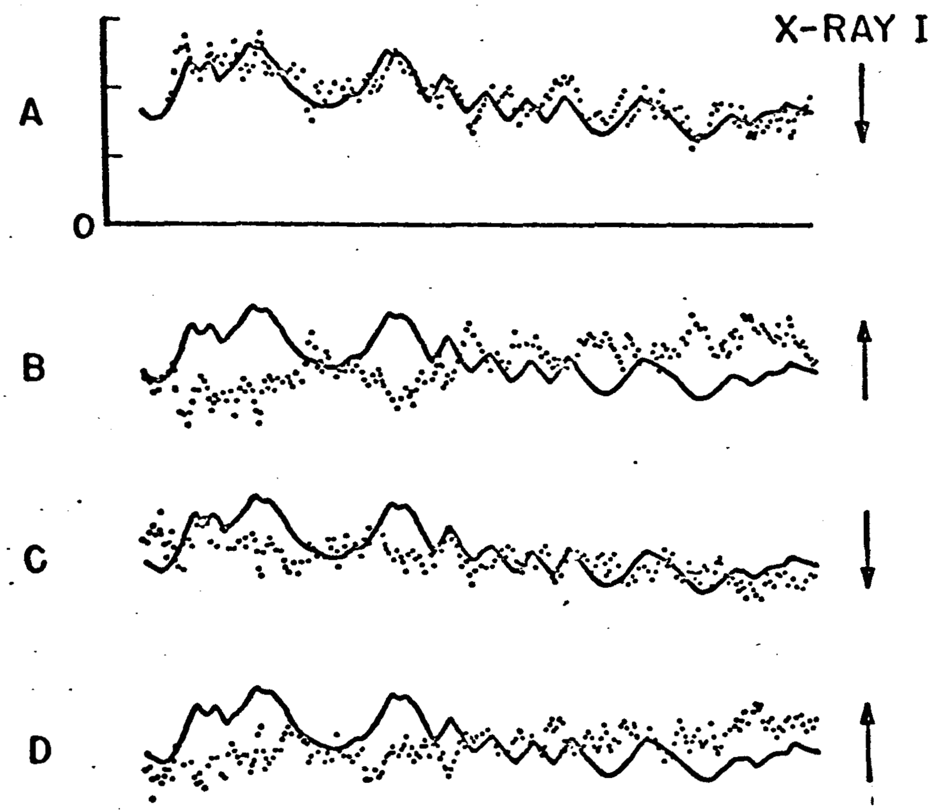

Fig. 1 IR signal (solid line) and x-ray signal averaged over 400 scans (dotted line). The IR intensity increases upward and x-ray intensity increases in the direction shown by the arrows. The x-ray signals in a) and b) correspond to the wave length of Te L_{a_1} while in c) and d) they are background levels at 3.400Å.

fluctuations correspond to about 1 division peak to peak or ~ 52 ppm. The estimated standard deviation in all traces of Fig. 1 would correspond to 0.2 divisions with the counting rate and the time constant used. The statistical accuracy could be improved somewhat by still longer scanning times; however, improvement by a factor of 3 would require 60,000 seconds on both the line and the line plus background and probably represents a practical limit.

Variation in spectral distribution of cathodoluminescence with carrier concentration in GaAs has been investigated by Cusano[7] and Casey[4]. Casey observed that certain parameters such as the wave length of maximum intensity, width at half-maximum and the slope of the logarithm of the intensity on the low energy side can be used to indicate the carrier concentration. In the present work, measurements on regions of high and low IR intensity showed no difference in spectral distribution to within experimental accuracy (~30 Å). Also, the Te concentration determined by microanalysis appears to exceed the carrier concentration by ~ 50 ppm. Therefore, it appears that precipitation of Te as noted by Meieran[9] in similarly doped specimens of GaAs is partly responsible for the decrease in IR intensity observed.

This work has been accepted for publication in Applied Physics Letters.

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B. Electron Transport and X-Ray Production in Solids

Grant AF-AFOSR-496-66, Joint Services Electronics Program

D. B. Brown

During this report period a technique developed elsewhere[1,2] has been modified and developed. The problem is the understanding of the physical processes resulting when a 5-50 kev beam of electrons impinges on the surface of a specimen in the electron probe microanalyzer[3,4,5]. The method of attack has been as follows. A form of the Boltzmann equation for electron transport has been adapted from the work of Bethe et al.[6] to the boundary conditions of this problem. The equation has been set up for numerical solution on a computer. The principal result is a distribution in energy, position, and angle of electrons in the specimen. These results have showed promise in their ability to predict such things as the distribution in depth below the specimen surface of X-ray production, the fraction of electrons backscattered, and the energy distribution of backscattered electrons. It is expected that this approach will contribute to the understanding of electron transport under these conditions and to the technique of quantitative analysis in the electron probe microanalyzer.

We have been conducting a study of the theoretical formulae for electron scattering and retardation in an attempt to improve the physical model incorporated in the computer program. In addition, the program has been modified and improved in the following ways:

1) It has been adapted to the Honeywell 800 computer of the USC Computer Sciences Laboratory.
2) Its running speed has been significantly increased through more efficient programming.
3) The convenience of data input and output has been increased.
4) As electrons scatter through the specimen they eventually lose their original collimation and become quite diffuse. The program has been modified so that it treats electron transport with the diffusion equation rather than the Boltzmann equation where this is permissible. The use of a diffusion equation increases running speed (because of a reduction in the number of variables treated) and leads to less trouble with difference equation instability.
5) The form of the Boltzmann equation used implies that most scattering will be through small angles. This approximation is

deficient for scatterers of low atomic number during the early stages of electron travel. The program has been modified to include a more elaborate and more correct scattering mechanism during these early stages.

6) As the electrons fall in energy toward the excitation energy of the X-ray line being considered, some account should be taken of the fact that energy straggling results in a distribution of electron energies around the calculated average; and, as a result, some electrons are more and some less effective as X-ray generators. A partial account of this effect has been included in the program.

Castaing[3] has defined f(x), and absorption correction function, as the ratio of the x-radiation which escapes the specimen in a given direction to that which would have escaped in the absence of absorption. This function has been measured for X-rays, and it has been of interest to know whether the same absorption corrections can be applied to the production of recombination radiation in GaAs in the electron probe. A calculation using the above transport equation program indicates that the X-ray curves are indeed a good approximation to those applicable to this optical excitation.

A key tool in quantitative analysis with the microprobe is the calibration curve relating the weight fraction of an element A in an alloy to ratio: the intensity of an X-ray line of A from the alloy/ the intensity of the same line from a specimen of pure A. Preliminary efforts to calculate such calibration curves using the transport equation program have been encouraging.

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C. Dielectric Constants and Infrared Absorption of GaSe

Grant AF-AFOSR-496-66, Joint Services Electronics Program

W. G. Spitzer, P. C. Leung, G. Andermann

In recent years the III-VI compound semiconductors such as GaSe, GaS, and GaTe have attracted considerable attention. Most of the studies have been concerned with the crystal structure, the electrical transport properties, and the optical properties in the vicinity of the fundamental absorption edge. In the present paper infrared measurements of GaSe are presented for the single phonon and some multiphonon lattice absorption bands, the high frequency dielectric constant and the low frequency dielectric constant. In addition the low frequency dielectric constant was determined by direct capacitance measurements of a thin specimen in a fully depleted condition.

The GaSe is a layer structure in which each layer consists of four parallel planes. The outer planes are close packed Se layers and the inner planes are close packed Ga layers[1]. The bonding between each of the 4 fold layers is predominantly Van der Waals although it has been suggested that there may be a small ionic contribution[2] to the stacking energy. There is general agreement that the 4 fold layer appears as in Fig. 1, where the bonding has been described[2] as tetrahedral sp^3 for Ga^{-1} and trigonal p^3 for Se^{+1}. There is some uncertainty as to the stacking of the 4 fold layers with respect to one another. The line along the Ga-Ga bond is a triad axis and hereafter will be referred to as the c axis of the crystal. Because of the symmetry, the electrical and optical properties are expected to be isotropic when electric vector $E \perp c$.

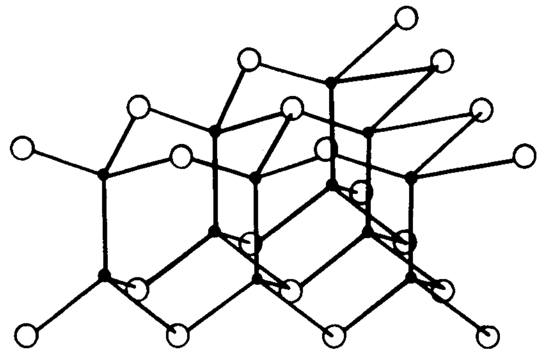
Early measurements of the absorption edge[3] placed the room temperature energy gap near 2.0 eV with a temperature dependence of $\sim -3.6 \times 10^{-4}$ eV/°K. However, other measurements[4] showed structure on the leading edge similar to that seen in a number of III-V compound semiconductors and generally atrributed to exciton absorption. Later observations[5] indicated that the structure could be increased by cold working the material and the possibility of other defect absorption has also been suggested. The most recent work has interpreted[6,7] the absorption edge as due to direct-forbidden transitions and indicated that the structure is due to exciton formation but where the carriers are regarded as remaining within a layer[8].

Very little information is contained within the literature concerning the values of the optical and static dielectric constants or the infrared lattice absorption of GaSe. Bube and Lind[9] reports some unpublished work of H. E. MacDonald in which values were given of $\epsilon_0 = 7 \pm 7$ (static value) and ϵ_∞ ranging from 4.7 to 3.4 depending upon sample purity. These authors also indicated that thermal and photoionization measurements for a level indicated a Franck-Condon shift which was large compared to the shifts in CdS. It was observed that this was in agreement with the lower value of ϵ_∞ for GaSe relative to that for CdS ($\epsilon_\infty \sim 6.3$). However, other work[8] indicates a room temperature reflectivity of ~21% for photon energies between 1.8 and 1.0 eV. This corresponds to an $n_\infty = 2.7$ or $\epsilon_\infty = 7.3$, and it seems unlikely that the dispersion from the absorption edge could be large enough to account for the difference between the quoted values. The $\epsilon_\infty = 7.6$ was used, in the latter case, in the estimation of the exciton binding energy; however, the energy calculated is substantially greater than the experimental estimate.

Single crystals were grown by the method of Bube and Lind[9], with similar electrical properties. Samples were prepared by cleavage of very thin layers from the bulk single crystal surface. After some practice it was possible to obtain areas of constant thicknesses several mm on a side and as thin as 1μ. Since ordinary Hall measurements were found to be unreliable because of difficulty in contacting the material properly, the carrier concentration was obtained from the same measurements used to determine the dielectric constant.

Since many measurements were to be made with Au electrodes, it was important to understand the Au-GaSe contact. It was found that the Au formed a blocking contact on the p-type GaSe, and the I-V characteristic indicated a barrier height of 0.52 eV. The photoresponse of a similar sample with a very thin gold electrode is shown in Fig. 2. From this plot a barrier height of 0.52 eV is also obtained[10].

Capacitance samples were made on cleaved layers 1-2μ in thickness, as measured by the infrared channel spectrum. On one entire side of the layer, a Au film approximately 1000 Å thick was deposited by vacuum evaporation. On the opposite side Au electrodes approximately 1 mm square were likewise evaporated. By applying a reverse bias to the small electrode, the depletion layer could be expanded. Figure 3 shows the capacitance and effective parallel loss conductance of a sample 1.75μ thick. As the bias is increased the capacitance decreases in the normal manner due to the widened depletion layer. The loss decreases due to the smaller amount of loss material (undepleted GaSe) through which the displacement current must flow. At approximately 2.5 V, the depletion layer reaches the back electrode, the capacitance becomes independent of bias, and the loss drops to a low, constant value. From this punched-through value of the capacitance and the sample dimensions the low frequency dielectric constant was found to be 8.0 ± 0.3. Using the voltage required for punch through plus the barrier height and the sample thickness, the net acceptor concentration was calculated to be 3×10^{15} cm^{-3}. At voltages lower than punch through, the capacitance and loss were both strong functions of frequency, suggesting an equivalent circuit more complicated than the simple parallel RC given by



ONE 4 FOLD LAYER OF GaSe WHERE Ga AND O Se

Figure 1

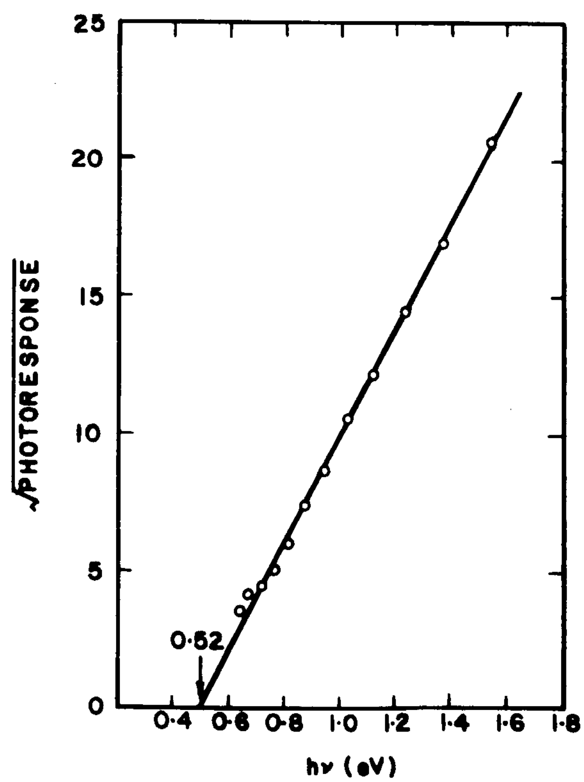


Figure 2. Square root of photoresponse per incident photon (arbitrary units) vs. photon energy, indicating a barrier height of 0· 52 eV; sample was Au on p-type GaSe.

the bridge. This behavior is to be expected of an un-depleted layer in series with the depleted layer. However, because of this effect it was possible to obtain meaningful C vs. V curves without fully depleting the sample. Several samples were measured at $4.2^{o}K$ where it was hoped that the carriers could be frozen out. Although an increase in the resistivity of approximately an order of magnitude was observed, this was still not sufficient to obtain meaningful capacitance data without punch through.

The room temperature reflectivity at near normal incidence was measured for a sample with the c axis in the surface. The measurements shown in Figs. 4 and 5 were made on the same sample surface with radiation polarized $E\parallel c$ and $E\perp c$ from $\nu = 1200\ cm^{-1}$ to $\nu = 216\ cm^{-1}$. The results were found to vary ~10% of the reflectivity between different samples largely due to the difficulties in surface preparation. While better large area surfaces with the c axis normal to the surface were prepared by cleaving, they would permit only $E\perp c$ measurements. These measurements were in agreement with those indicated above. As discussed in the previous section, the electrical ϵ_0 measurements could be made only for the $E\parallel c$ case.

The reflectivity data of Figures 4 and 5 are fitted in each case by using the single frequency classical dispersion model with frequency independent damping. This model has been discussed extensively by a number of authors[11]. The optical constants, n and k, are calculated as functions of ν from the damped oscillator model by assuming values for ϵ_∞, ν_0, γ, and $4\pi\rho$, where ν_0 is the resonance frequency which corresponds to an infrared active transverse optic phonon of near zero wave vector, γ is the frequency

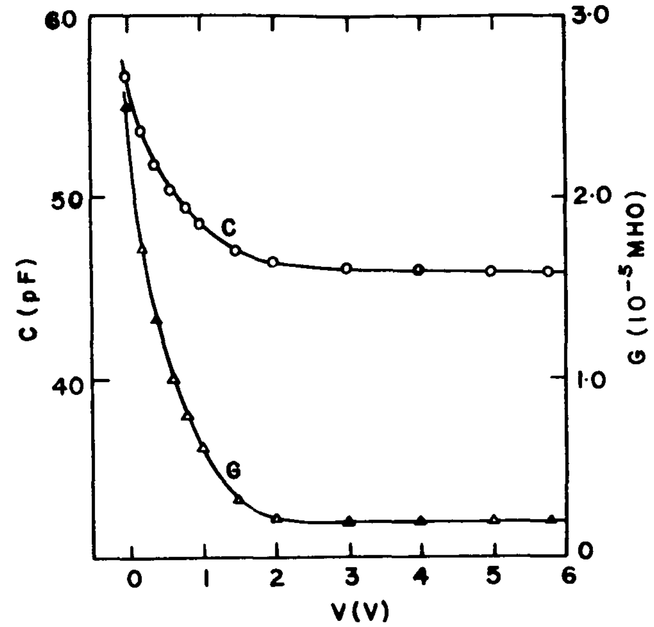


Figure 3. Capacitance and effective parallel conductance of Au-GaSe-Au sample exhibiting punch-through (see text).

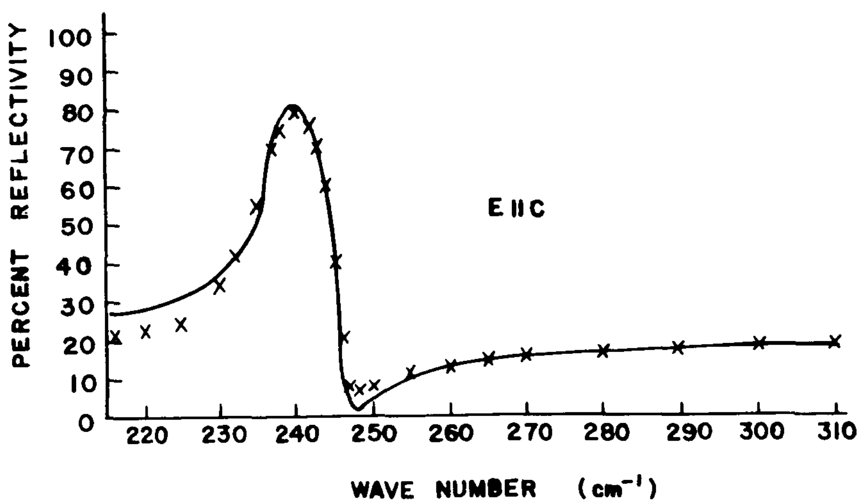


Figure 4. Room temperature reflectivity of GaSe with E$\|$C. The crosses are the experimental points, and the curve is the reflectivity calculated with the dispersion parameters of Table 1.

independent damping constant, and $4\pi\rho$ is the resonance strength. The optical constants are then used to calculate the reflectivity. The best fits obtained to the data are shown by the curves in Figures 4 and 5 and the dispersion parameters given in Table 1. It may be noted that while

TABLE 1

	$E \perp c$	$E \parallel c$
ϵ_∞	8.4	7.1
ν_0	230.7 cm^{-1}	237.0 cm^{-1}
$4\pi\rho$	1.8	0.54
γ	2.0 cm^{-1}	1.8 cm^{-1}
ϵ_0	10.2	7.6

resonances occur at close to the same frequency, the $4\pi\rho$ for $E \parallel c$ is less than 1/3 of the value for $E \perp c$. Because of the geometry of the optical system there was a small component of $E \perp c$ in the $E \parallel c$ measurement (for the extreme rays E made an angle of 12° with the c axis). Reduction of this $E \perp c$ component by a factor of 2 produced no significant change in the reflectivity for the $E \parallel c$ in Fig. 4. In addition, the $\epsilon_0 = 7.6$ is in satisfactory agreement with the value of 8.0 ± 0.3 obtained from the low frequency capacitance measurements.

As discussed by previous authors[11], while the fitting of the reflectivity data is capable of considerable accuracy, the limitation in the present measurements is the accuracy of the reflectivity measurements themselves.

For example a change in R_∞ (reflectivity for ν where $\epsilon = \epsilon_\infty$) from 23.7% to 21.1% changes ϵ_∞ from 8.4 to 7.3. Because of the previously mentioned difficulties in surface preparation the dispersion parameters for E|c were also determined from channel spectra. In this case the transmission of a 7.06×10^{-3} cm sample with cleaved surfaces was measured from 1356 to 320 cm^{-1} and 41 fringe maxima were observed. The frequencies of the maximum points are indicated in Fig. 6. The method used to fit the fringes was the same as one previously described[12] but with only the lattice term taken into account, i.e.

$$n^2 = n_\infty^2 + \frac{4\pi\rho}{1 - (\nu/\nu_0)^2} .$$

The dispersion from the absorption edge is neglected since the largest frequency used here is more than an order of magnitude less than the edge frequency. The free carrier susceptibility term is omitted as samples of similar electrical properties and of considerably greater thickness shown no evidence of free carrier absorption. This is also consistent with the electrical measurements which indicated carrier concentrations $\sim 10^{15}$ cm^{-3}. The dispersion parameters used to obtain the calculated curve of Fig. 6 are

$$n_\infty^2 = 7.45, \quad 4\pi\rho = 2.35, \quad \nu_0 = 230.7 \text{ cm}^{-1} ,$$

and

$$\epsilon_0 = 9.80 .$$

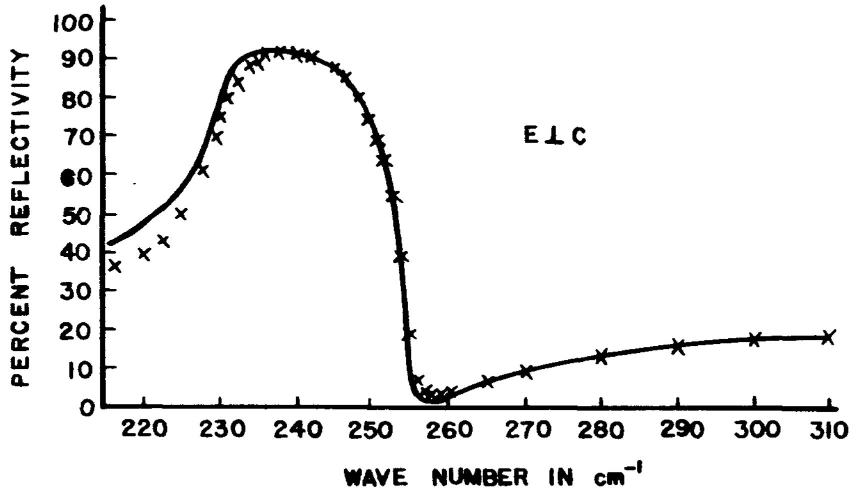


Figure 5. Room temperature reflectivity of GaSe with E⊥C. The crosses are the experimental points, and the curve is the reflectivity calculated with the dispersion parameters of Table 1.

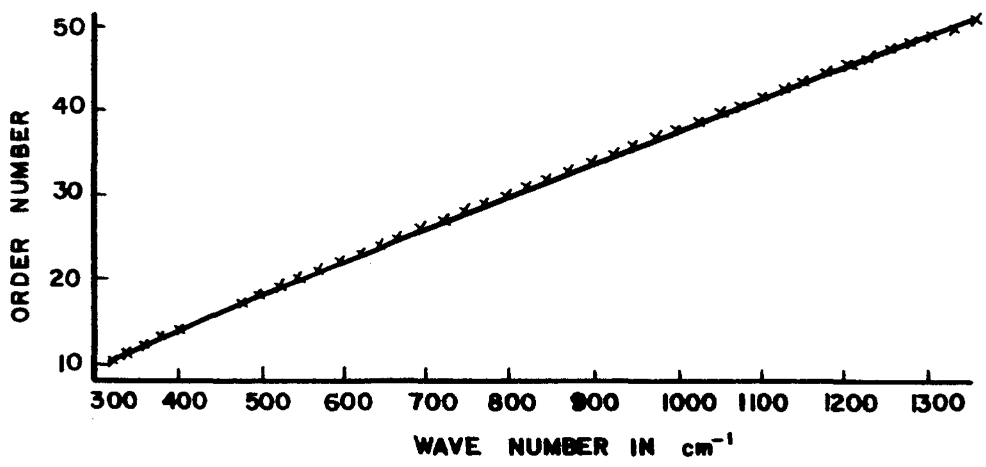


Figure 6. The points are the fringe maxima as measured for a $7{\cdot}06 \times 10^3$ cm thick sample with E⊥C. The curve is calculated as described in the text.

These are regarded as more accurate values than those of Table 1 for E|c.

Figures 7 and 8 show the transmission data for a number of samples of different thickness. All of the data of Figure 7 are for E$\perp$ c and all samples but the thickest one had cleaved surfaces. The thickest sample had polished surfaces with the c axis in the surface. Figure 8 shows the comparison of the E$\|$ c and E$\perp$ c curves for this same sample. It may be noted that because of the lower value of $4\pi\rho$ for E$\|$c (see Table 1) the two curves of Fig. 6 show considerable difference in absorption between 300 and 400 cm^{-1}. The transmission for both polarizations indicate the presence of several absorption bands between 400 and 510 cm^{-1}. These are probably combination bands involving the optical branches of the phonon spectrum as they occur at frequencies in the range near $2\nu_0$. To attempt an assignment of phonon energies from the absorption measurements would require a more detailed study than that presented here.

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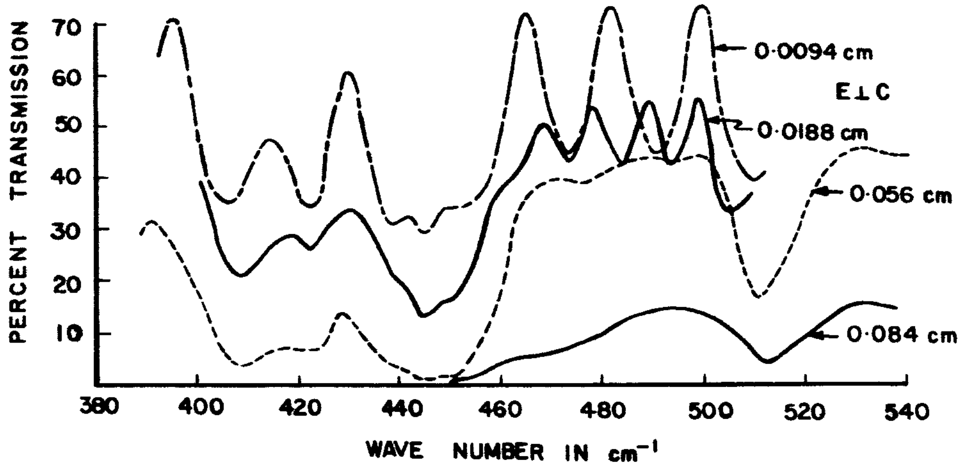

Figure 7. Transmission of samples of different thickness for E⊥C.

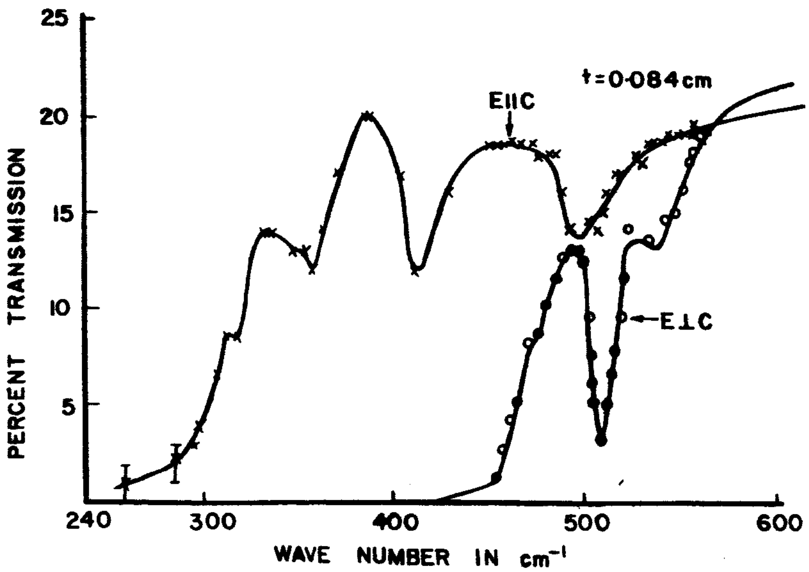

Figure 8. Transmission of a sample with E⊥C and E∥C.

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D. Local Mode Absorption of Al and P in GaAs

Grant No. AF-AFOSR-496-66, Joint Services Electronics Program

W. G. Spitzer, O. G. Lorimor

There have been a number of recent experimental and theoretical studies of infrared absorption by local vibrational modes in crystals[1]. Two of these investigations were directed toward the study of lithium modes in GaAs[2] and substitutional Al and P in GaSb[3]. In the Al and P cases each impurity introduced a single absorption band. No evidence for the first overtone ($n = 0 \leftrightarrow n = 2$) transition was observed. In the present work we wish to report the observation of the $n = 0 \leftrightarrow n = 2$ transitions for both Al and P impurities in GaAs.

Figure la and lb show the bands due to the Al and P respectively while the sample is in contact with a liquid nitrogen cooled mount. Between liquid nitrogen and room temperature the absorption peak positions shift

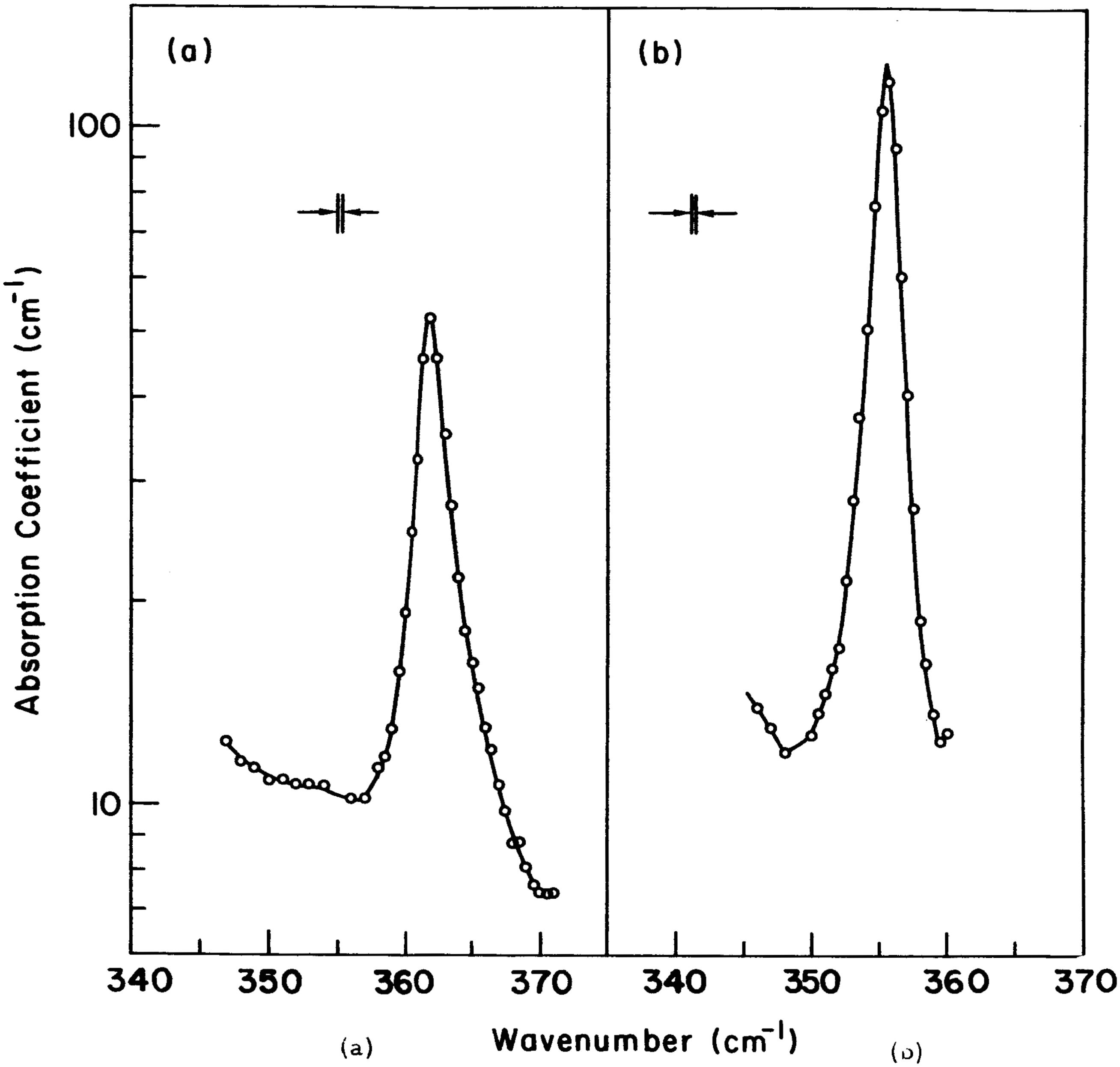

Fig. 1. Liquid nitrogen temperature absorption peaks due to substitutional impurities in GaAs; a) Sample contains 69 parts per million of aluminum; b) Sample contains 140 parts per million of phosphorus.

from 362 cm^{-1} to 359 cm^{-1} for the Al and 355 cm^{-1} for P. Similar shifts were reported for these same impurities in GaSb.

Al and Ga have the same tetrahedral covalent radii (1.26 Å), and hence, following the arguments of Hayes[3], we expect the 362 cm^{-1} local mode frequency to be close to the optical mode frequency in AlAs. The best figure available for the latter is 356 cm^{-1}.[4] The P radius (1.10 Å) is also close to that of As (1.18 Å) and the local mode frequency of 353 cm^{-1} compares with the transverse optic (near $\bar{q} = 0$) value of 366 cm^{-1} for GaP. The observation that the local mode frequency is below 366 cm^{-1} is in agreement with the measurements of Chen and Pearson on $GaAs_{1-x}P_x$.[5]

Local mode frequencies of 374 and 368 cm^{-1} for Al and P in GaAs are calculated by using the theory of Dawber and Elliott[6]. Although the theoretical calculations are based on the silicon phonon dispersion curves, the single phonon density of states curves for Si and GaAs are very similar[7]. The agreement here is as good as the agreement for the boron local mode frequency in boron-doped silicon[8].

Figure 2 shows the height of the absorption band at liquid nitrogen temperature as a function of the impurity content. Many of the Al samples and both of the P samples were lithium diffused to effect electrical compensation. However, in neither case is the height of the band correlated with the lithium content. The encircled point is for a sample which was not lithium diffused. The impurity concentrations were obtained from spark source mass spectrometer analysis[9] of samples from the same portion of

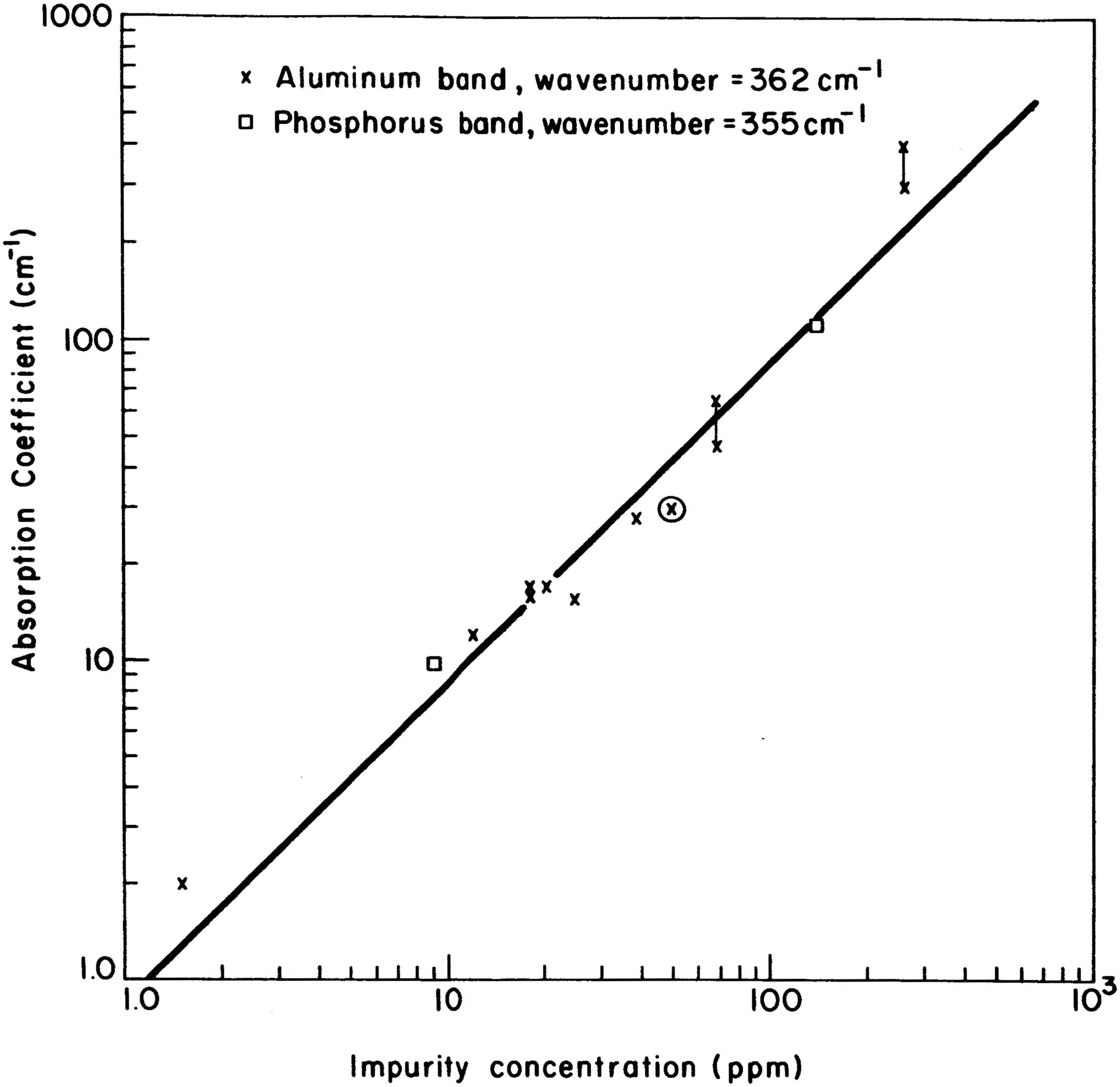

Fig. 2. Height at liquid nitrogen temperature of aluminum and phosphorous absorption bands as a function of concentration. The encircled point represents a sample which contained no lithium.

the ingot as those used for the optical measurements. Note that the points for both impurities fall close to a straight line of slope 1, in agreement with the theory of Dawber and Elliott. Since the half-widths of the Al and P bands are very nearly the same, the $\frac{1}{N}\int \alpha \, d\upsilon$ is the same for both impurities where N is the impurity concentration. Measurements of these bands can be used to detect impurity concentrations as low as $\sim 5 \times 10^{16}\ \mathrm{cm}^{-3}$.

With tetrahedral symmetry the $n = 0 \quad n = 2$ overtones are symmetry allowed and have been previously observed for B in Si[10]. In the present case bands are present at 722 cm^{-1} (liquid nitrogen temperature) and 718 cm^{-1} (300°K) in the Al doped samples and at 709 cm^{-1} (liquid nitrogen temperature) in a P doped sample. The ratio of the height of the absorption coefficient of the local mode band to the first overtone band is $\frac{\alpha_{n=0-1}}{\alpha_{n=0-2}} \sim 1.2 \times 10^{3}$ for P and 4×10^{2} for Al.

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E. III-V Compound Semiconductors and Their Alloys

Grant AF-AFOSR-496-66, Joint Services Electronics Program

J. M. Whelan

Experimental facilities are in the construction stage for the growth of epitaxial films, purification of reagents used for their synthesis, and measurement of the optical properties of the films. Some delays have been experienced in developing improved techniques for minimizing contamination sources in the growth equipment. An apparatus utilizing fiber optics has been designed for measuring optical absorption coefficients and photo luminescence. The critical requirement of the fiber optics is that the divergence of a transmitted beam be as small as possible. Suitable polishing techniques have been developed for fiber bundles with sloped ends to match the sample sizes and spectrometer slit. For a bundle 36 inches long, the intensity of transmitted light 8° off axis is less than 50% of maximum for incident light having an 8° divergence. This compares with approximately 45° for commercial fiber bundles of equal length.

1.2 MAGNETISM

A. Ferromagnetic Resonance in Doped YIG Crystals

Grant AF-AFOSR-496-66, Joint Services Electronics Program

T. Hartwick, J. Smit

In this report period a new magnetic effect, magnetic annealing by electronic motion, has been identified in which the magnetic properties are affected by cooling the sample in a magnetic field. This effect, found in YIG crystals doped with Fe^{2+} ions, has permitted a good check to be made on the impurity energy level scheme which is difficult to determine in any other experiments. Other experiments in cobalt ferrite have shown that electron diffusion can produce magnetic annealing[1].

The last progress report discussed the ground state orbital doublet configuration which can occur for Co^{2+} or Fe^{2+} ions and the experiments indicated that this impurity level system did exist in crystals with both ions present. The present experiments on crystals with only Fe^{2+} ions, $Y_{2.8}Fe_{5.1}Si_{0.18}Co_{0.00}O_{12}$ by spectroscopic analysis, show clearly that Fe^{2+} ions have the orbital doublet configuration. In Figure 1 is shown the field for resonance H_R and line width ΔH as $T \rightarrow 0^{\circ}$. Although the resonance line shape is somewhat obscurbed by additional modes (probably magnetostatic modes) resulting in inaccurate ΔH, dependence on direction was not critical and, hence, not explainable by near energy level crossings[3]. Upon cooling in H_a just large enough to saturate the material, a large reduction in line width is found. It is concluded that the large ΔH value at $4.2^{\circ}K$

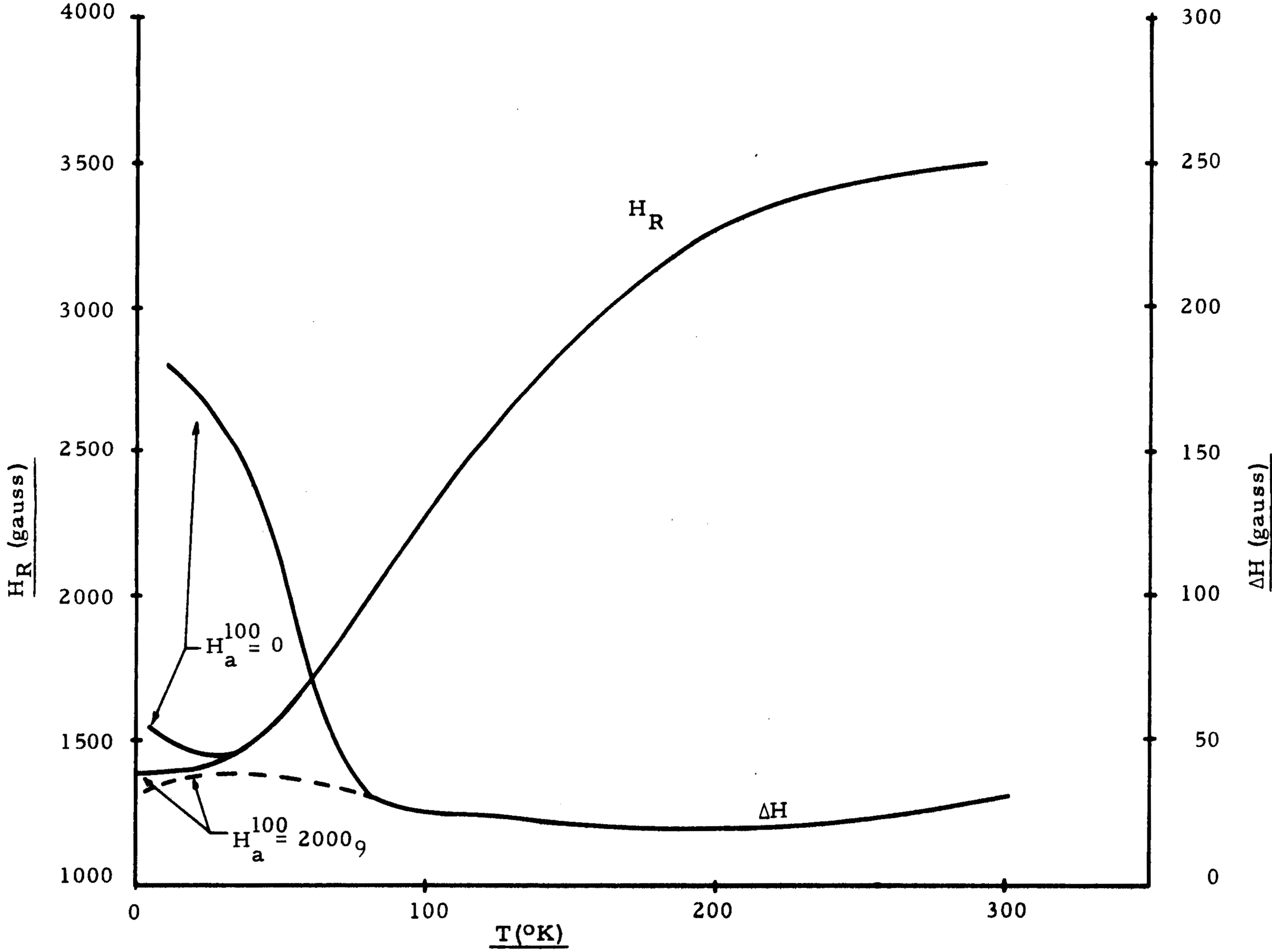

Figure 1. Graph of field for resonance H_R and linewidth ΔH versus temperature for a Fe^{2+} doped YIG sphere. The data is taken at 10,002 mH_z with and without cooling in an applied field H_a^{100} directed along [100].

without H_a applied during cooling is obtained from a local magnetic ordering that reproduces the sphere domain pattern, effectively making the sample inhomogeneous. A more striking display of the annealing effect is seen in Figure 2 from the variation of H_R with θ in a (110) plane with 3 orientations of applied field H_a^{100}, H_a^{110} and H_a^{111}. For $H_a^{[111]}$ the uniaxial induced anisotropy becomes as large or larger than cubic terms. Some small error in H_R can be noticed and comes from slight de-annealing as the field direction changes in the course of making measurements.

An interpretation of this data can be made from the energy level configuration of the Fe^{2+} ion for which the variation of E with θ is shown[4] (Figure 3). There are 3 sets of curves corresponding to four inequivalent sites in YIG associated with the four body diagonals of the unit cell with two sites having an energy degeneracy in the (110) plane. (The correspondence is Site A - $[111]$, Site B - $[\bar{1}11]$, Site C - $[11\bar{1}]$, and Site D - $[1\bar{1}1]$). Slonczewski[4] has used the same configuration to explain the magnetic anisotropy and annealing in $Co_xFe_{3-x}O_4$. However, there the annealing takes place at $T \sim 400^\circ K$ and arises from Co^{2+} ion transfer to a site with lowest energy according to Figure 3. For example, $H_a^{54.7^\circ} = H_a^{111}$ would induce ion transfer to site A. In the present case the ion transfer also takes place but by electron transfer, e.g., $Fe^{+2}_{\text{site C}} + Fe^{3+}_{\text{site A}} \rightarrow Fe^{3+}_{\text{site C}} + Fe^{2+}_{\text{site A}}$. This explains why the annealing only occurs at low temperatures since electrons remain mobile after ordinary ion motion ceases.

The variation of H_R with θ can be deduced from the generalized

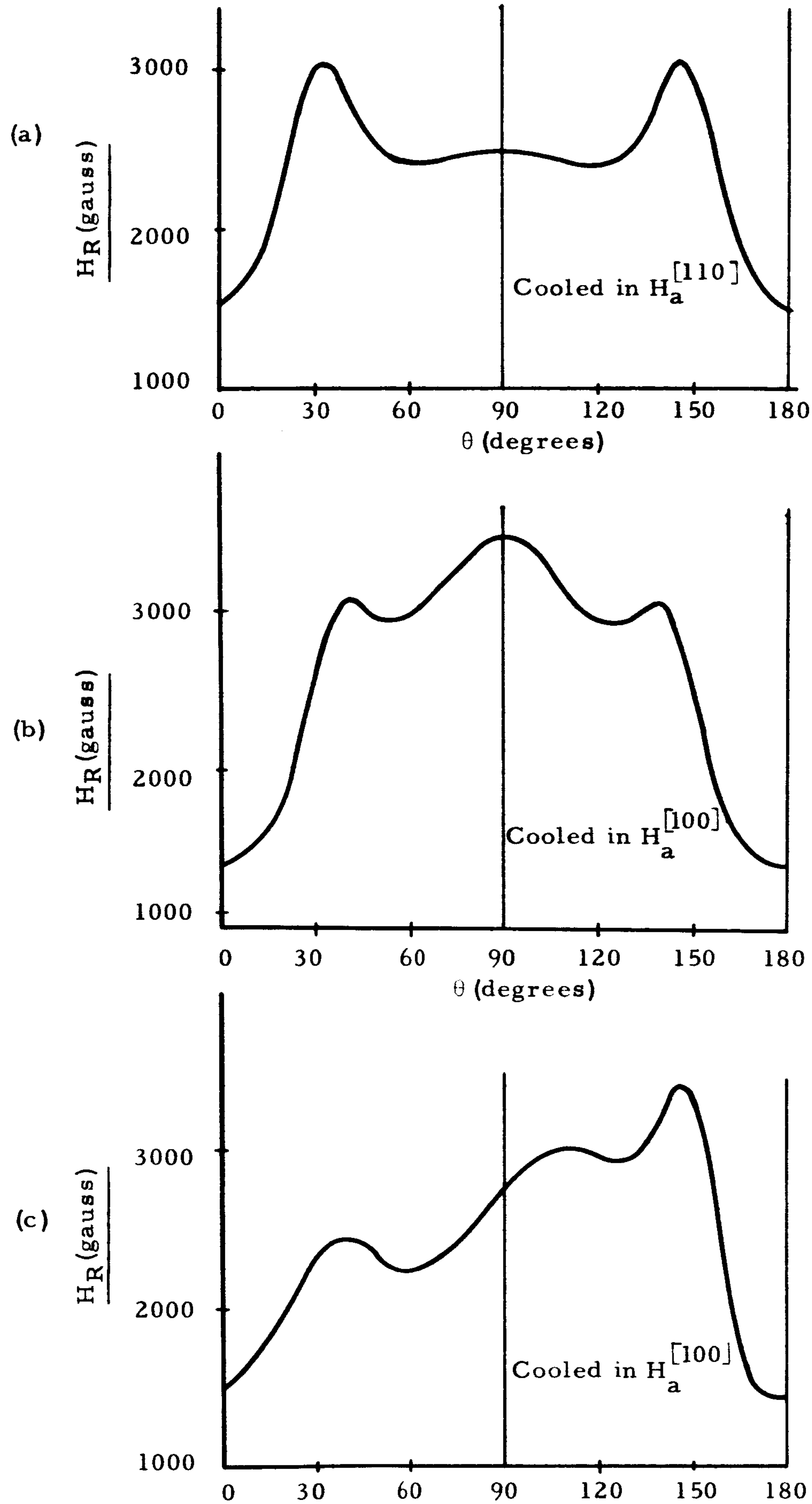

Figure 2. The variation of H_R with θ measured away from [100] in a (110) plane taken at 4.2°K at 10,002 mHz. The sample is cooled in a) $H_a^{[110]}$ b) $H_a^{[100]}$ and c) $H_a^{[\bar{1}11]}$.

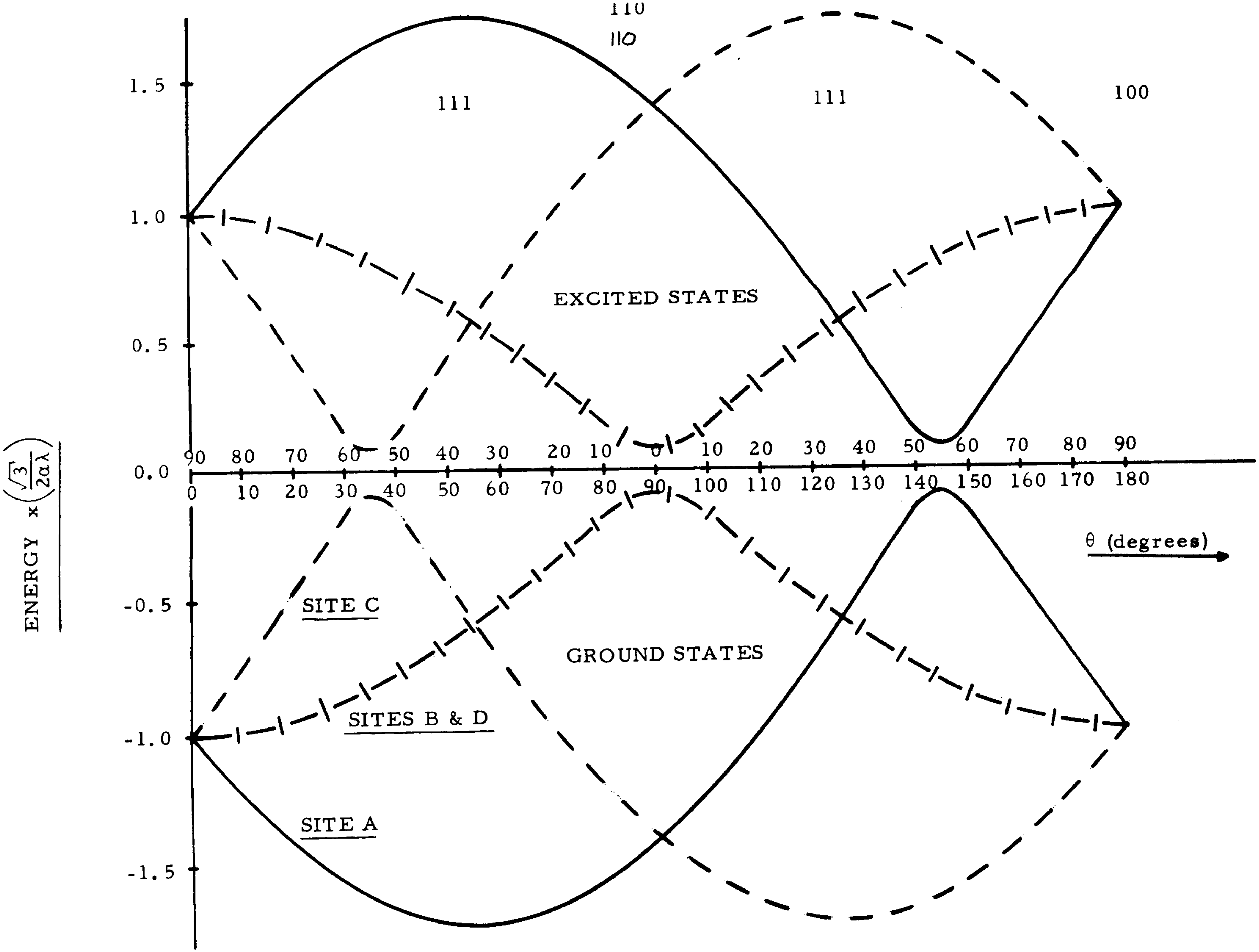

Figure 3. Theoretical variation of orbital doublet ground state energy E versus θ measured in (100) plane. The energy in normalized to the spin orbit coupling constant $\alpha\lambda$ and at level crossings the curve has been sketched in.

resonance expression[5], ignoring the smaller "pure YIG" anisotropy energy:

$$\frac{\omega r}{\partial} = \frac{1}{M \sin\theta} \left\{ \frac{\partial^2 [\vec{M} \cdot \vec{H} + E(\theta, \varphi)]}{\partial\theta^2} \right\}^{1/2} \left\{ \frac{\partial^2 [\vec{M} \cdot \vec{H} + E(\theta, \varphi)]}{\partial\varphi^2} \right\}^{1/2}$$

but very roughly H_R correlates with $-\frac{\partial^2 E}{\partial \theta^2}$. For H_a^{110} one expects the A and C sites to be populated and thus a large value of H_R at $\theta = 35.3^o$, 145.3^o coinciding with the sharp peaks in the $E(\theta)$ curve; similarly for H_a^{100} one expects the same two peaks from the A and C sites plus a large peak at $\theta = 90^o$ from the B + D sites all of which are observed in Figure 2B. Finally, for H_a^{111} ($\theta = 54.7^o$) a large peak is expected for only $\theta = 145.3^o$ as seen in Figure 2C. Good qualitative agreement is found but a quantitative check has not yet been possible due to the de-annealing and the inability to quickly quench from a fixed low temperature. At $T = 4.2^oK$ the characteristic time with which H_R^{110} decays is of order 20 minutes but the decay does not follow an exponential law.

The above experiments represent a good start toward understanding Co^{+2} and Fe^{+2} ions in YIG and may explain the curious increase in ΔH as $T \rightarrow 0^o$ for In^{+3} doped YIG[6]. Experiments will be continued in the next period to find more quantiative agreement and to measure Co^{2+} doped sample.

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B. Nuclear Magnetic Resonance in Ferromagnetic Alloys

Grant AF-AFOSR-496-66, Joint Services Electronics Program

S. Ogawa

NMR is a useful tool for the study of the internal magnetic fields of ferromagnetic compounds. The resonance frequency is determined by the magnitude of the static magnetic field at the nucleus. In the case of ferromagnetic materials this field appears to be much larger than the externally applied field and is of the order of several hundred thousand gauss. The value of this field varies in several factors for different atomic nuclei and in fractions for different states of materials. By comparing the experimental findings in slightly different states of the material, it is possible to get useful information about the magnetic state of the various constituent ions of the material.

a) As was reported before, the NMR frequencies of Mn and Cu nuclei in the Ferromagnetic Heusler Alloy (Cu_2MnAl) do not have the same temperature dependences. To get more information to solve this unexpected result the sign of the internal field at these nuclei was measured by applying an external magnetic field. Measurements were made at liquid nitrogen temperature to maintain a large signal to noise ratio since the resonance signal mainly comes from the domain walls which become swept away as the external field is applied. The signs of the internal field for both isotopes were negative, that means increasing external magnetic field decreases the resonance frequency. The intensity of the resonance line has a maximum at a field of about two thousand gauss which is related to the change in the reversible permeability (see Figure 1). A similar study for nonmagnetic atoms such as Cu, Al and Si surrounded by ferromagnetic neighbors has been started in order to study the interaction mechanism in magnetic alloys and compare this with the result for the Heusler Alloy.

The following equipment for preparing various kinds of alloys has been constructed: an induction melting system, a tube furnace for annealing in vacuum or hydrogen atmosphere, a vacuum system which extends down to 10^{-6} mm Hg for the reduction of oxides, and a grinding machine to make a fine powder of alloys of around 100 microns particle size. The resonance frequencies for Ni and Fe nuclei in their alloys fall between 20 Mc to 50Mc. The resonance of Fe^{57} in pure sponge powdered iron has been observed at room temperature with the new spin echo apparatus.

b) The Europium chalcogenides and its derived compounds with another rare earth element are being studied. The separate contributions of conduction and core electrons will be obtained from the investigation of the

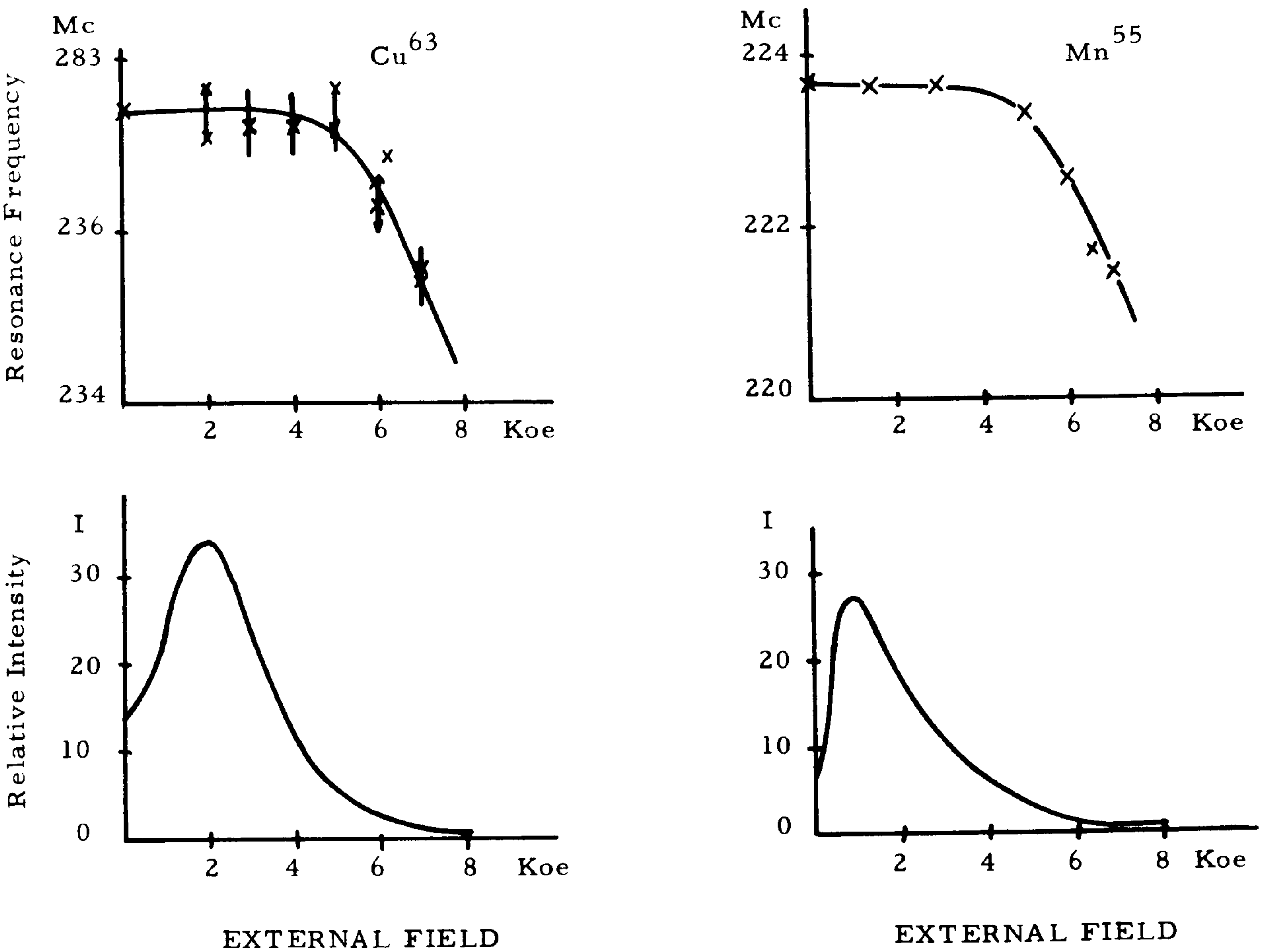
Mc
283
236
234
Resonance Frequency
Cu^{63}
2
4
6
8
Koe
Mc
224
222
220
Mn^{55}
2
4
6
8
Koe
I
30
20
10
0
Relative Intensity
2
4
6
8
Koe
EXTERNAL FIELD
I
30
20
10
0
2
4
6
8
Koe
EXTERNAL FIELD

Figure 1

internal field of the rare earth atoms. These compounds have low curie temperature (EuO 70°K, EuS 16°K, EuSe 6°K). Accordingly a special dewar which has a finger at the bottom to contain the sample, was made. The coils for transmission and detection of radio frequency power are attached outside of the finger and cooled with liquid nitrogen bath.

A tentative experiment was done with five grams of EuS, but expected resonance line has not been observed. The failure of this tentative experiment is considered to be caused partly by 1) material which is not stoichiometric and which shows weak characteristic lines by x-ray diffraction and 2) partly by insufficient radio frequency power transmission.

1.3 CRYSTALS

A. Defect Chemistry of CdS

Grant AF-AFOSR-496-66, Joint Services Electronics Program

F. A. Kroger, S. Prussin

The attempts to measure both the Hall effect and the resistivity of CdS crystals in equilibrium with cadmium vapor at high temperature have been continued. Improvement of the method of maintaining well-defined cadmium pressures for a long time as described in the previous progress report has made it possible to keep a crystal in the apparatus under cadmium pressure and carry out measurements on it for several weeks.

Measurements at $800^{o}C$ show a conductivity independent of the cadmium pressure from 0.1 - 1 atm, indicating the presence of donors in concentrations varying from $3 \times 10^{17} cm^{-3}$ in crystals from one batch (R) to $3 \times 10^{16} cm^{-3}$ for crystals from another batch (E.P.). The latter represents the purest CdS available to date. The concentrations given agree closely with the concentrations of foreign donors (mainly Al) found in the crystals by mass spectrographic analysis.

At lower temperatures (700, $600^{o}C$) lower electron concentrations and conductivities are found, varying with the cadmium pressure $\alpha\ p_{Cd}^{m}$, with $m \approx 4$. This suggests precipitation of the foreign donors. Other authors, observing a similar behavior, have attributed it to the formation of native interstitial Cd donors. This is probably not correct. Heating in vacuum at $T \geq 400^{o}C$ leads to a quick loss of the conductivity (as also found by other workers). It was concluded initially that this indicated that equilibrium throughout the sample was quickly reached. This proved to be not correct; the loss of conduction is due to the formation of a thin high-resistivity surface layer, which makes it difficult to pass current through the sample even when the inside of the crystal is still conducting. Times needed to reach equilibrium throughout a crystal of thickness ≈ 1 mm may be as long as 24 hours at 600^{o}C and 7 hours at 800^{o}C. This makes it necessary to repeat the experiments under conditions which make it possible to check whether equilibrium is reached. Work along these lines is in progress.

B. Ionic Conductivity in High-Resistivity CdS

Contract AF-AFOSR-986-66, Directorate of Chemical Sciences

F. A. Kroger, D. Yuan

CdS prepared in cadmium vapor of varying, relatively high activity ($p_{Cd} \geq 0.01$ atm) is an n-type semiconductor. When prepared under a lower Cd pressure it is a high-resisitivity semiconductor or even an insulator. Attempts will be made to show the presence of small ionic contributions to the conductivity by means of high-temperature measurements on a cell of the type

$$C, S_2 / CdS / C, Cd$$

which should given an emf equal to

$$E = -t_i \frac{\Delta G}{nF}$$

t_i being the ionic transport number, F the Faraday number, n the number of charges carried by the migrating defect, and ΔG the free enthalpy of the reaction

$$Cd + S_2 \rightarrow CdS \; ; \quad \Delta G$$

Apparatus for this type of measurement is being constructed.

C. Optical Properties of Bromine-Rich AgBr

Contract AF-AFOSR-986-66, Directorate of Chemical Sciences

F. A. Kroger, P. B. P. Phipps

A pumping system has been built and an apparatus has been constructed which makes it possible to study the optical absorption and the photoconductivity of AgBr in equilibrium with a variable bromine pressure at temperatures up to the melting point. The goal is to determine in this manner the concentrations of free holes and non-ionized native acceptor centers (V_{Ag}^{x}).

A Zeiss double monochromator has been purchased; an optical bench and stands for light source and detector are under construction.

D. Theory of Metallic Binding

Grant AF-AFOSR-496-66, Joint Services Electronics Program

L. Kleinman

The first in a series of papers on the theory of metallic binding was completed and accepted for publication by The Physical Review. We demonstrated that the orthogonalized plane wave method yields binding energies, equilibrium lattice constants, and bulk moduli for metals (aluminum in particular) in as good agreement with experiment, as do the cellular methods. The failure of other workers to achieve good agreement is due to the use of incorrect core eigenvalues; one must also be careful not to double count certain interactions. There appear to be uncertainties in the exchange and correlation contributions to the energy much larger than the error in the calculated binding energy (for either OPW or cellular calculations). The probable effects of eliminating some of these uncertainties with a screened Hartree-Fock crystal calculation were discussed in an appendix to the paper and we have initiated the first complete self-consistent Hartree-Fock crystal calculation which when completed will put crystal calculations on a par with atomic calculations.

Work on pseudopotential calculations of the energy bands of SnTe, Bi and Sb was initiated. The electrostatic contribution to inner displacements of atoms in strained exagonal close packed metals was completed and the band contributions are now being calculated. Calculations of the phonon dispersion curves of Al using a one parameter pseudopotential were initiated.

1.4 SUPERCONDUCTIVITY

A. Type II Superconductivity

Contract AT(11-1)-113#16, Atomic Energy Commission

M.A.R. LeBlanc, D. J. Griffiths, B.C. Belanger, C.T.M. Chang,

H.G. Mattes

a) Evidence for the existence of a surface sheath capable of supporting appreciable induced currents throughout the mixed state in nonideal type II superconductors has been obtained. The existence of such a surface sheath has been predicted recently[1] from solution of the Ginzburg-Landau equations in fields lower than the upper critical field H_{c2}. The initial portion of minor hysteresis loops in wires of NbTa is seen to exhibit complete flux shielding or flux trapping over an appreciable range of change of axial field. From these observations we can also deduce that, in a semi-reversible type II superconductor which cooled through T_c in the presence of a stationary longitudinal magnetic field, a macroscopic current is flowing in this surface sheath and a macroscopic diamagnetic current is flowing in the bulk of the specimen.

b) The concept of the critical state due to Bean, Kim and Anderson has been very useful in understanding the magnetic behavior of nonideal type II superconductors. A detailed study of the magnetization curves of NbTa as a function of magnetic field and temperature history reveals significant deviations from the predictions of this model. In particular, as shown in Figure 1, the remanent magnetization versus the maximum field impressed

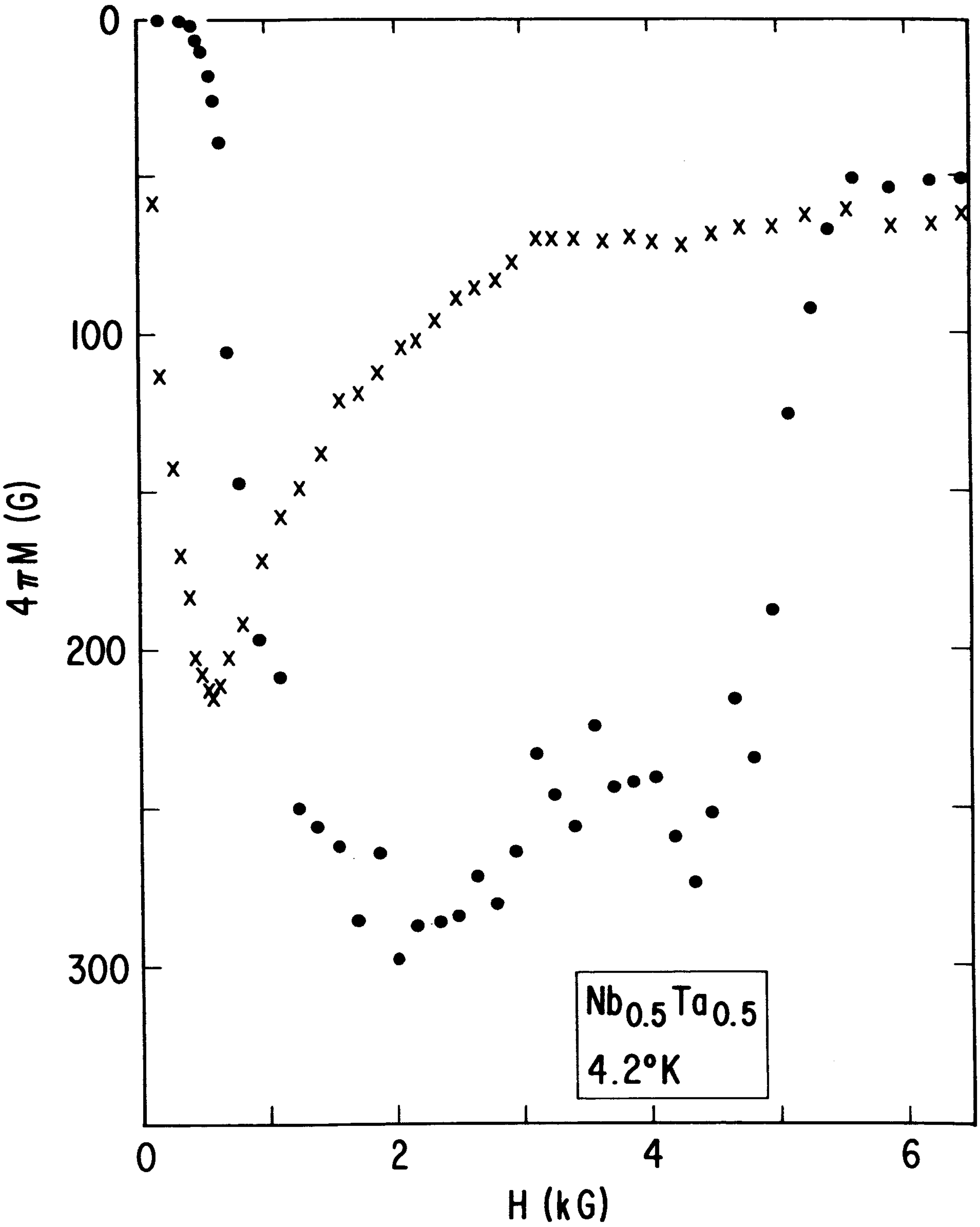

Figure 1

on the sample is found to exhibit resonance-like behavior. Further, the remanent magnetization is seen to depend on the procedure followed in trapping the flux in the specimen. According to the critical state model, the remanent magnetization should attain a saturation plateau. The value for this plateau should be independent of procedure for a given specimen and temperature. Our work indicates that this model must be modified. We can qualitatively interpret our observations by retaining the critical state picture but adding a feature originally suggested by Saint James and de Gennes. We suggest that superconductivity persists above H_{c2} or first appears below H_{c3} in isolated internal surfaces as well as at the physical surface of the specimen. These internal nucleation sites possibly correspond with grain boundaries. In these regions, as in the surface sheath, flux exists in continuous rather than in discrete form. The thickness, the order parameter and the diagmagnetism of these regions increase as $\langle B \rangle \overset{\approx}{<} H_{c1}$. These nucleation sites are inoperative in a sample which cools through T_c in $H = 0$ when the applied field remains less than H_{c2}, hence flux is more readily pinned by imperfections down to zero field. However, superconductivity first appears in these regions in a sample which is allowed to cool through T_c in $H_{c1} \lesssim H \lesssim H_{c3}$. Thus flux is expelled from the specimen due to the Meissner pressure of these regions when $\langle B \rangle$ is lowered below H_{c1}.

c) We have found that a very nonideal type II superconductor (e.g. Nb_3Zr) exhibits considerable critical current degradation for currents of very low frequency but of varying polarity in static longitudinal magnetic fields[3].

Figure 2 shows the critical current, denoted I_{CA}, observed when the current is increased continuously until quenching occurs. The critical current, denoted I_c', is observed when, previously, the conduction current was cycled once in the opposite direction to a value near I_{CA}. This procedure is shown schematically in the inset of Figure 3. The degradation of the critical current which appears when the polarity of the current during the cycle is opposite to that subsequently impressed is seen to depend on the amplitude I_M of the cycle. Figure 3 shows the typical dependence of I_c' on I_M. It is clear that I_M must exceed a threshold value, denoted I_T, in order that I_c' be degraded with respect to I_{CA}. The open circles of Figure 2 give the threshold current determined at various fields.

The solid curve of Figure 3 shows the result expected from the model we have proposed to account for our observations. According to this model, a substantial fraction of the conduction current flows in a surface sheath. In this surface sheath, magnetic flux exists in "continuous" form rather than in "discrete" filaments. Thus the concept of flux pinning does not apply to this surface sheath and the self-field configurations associated with currents $I < I_T$, can adjust smoothly as the polarity of the current alternates. However when I exceeds I_T, a helical configuration of flux is established in the core or bulk of the wire. Some or most of this helical flux in the core remains trapped or pinned as the current is removed since the specimen seeks to maintain the existing flux configuration. As a current of opposite polarity is increased, a destruction of this pinned flux must occur and can lead to premature quenching. According to this picture,

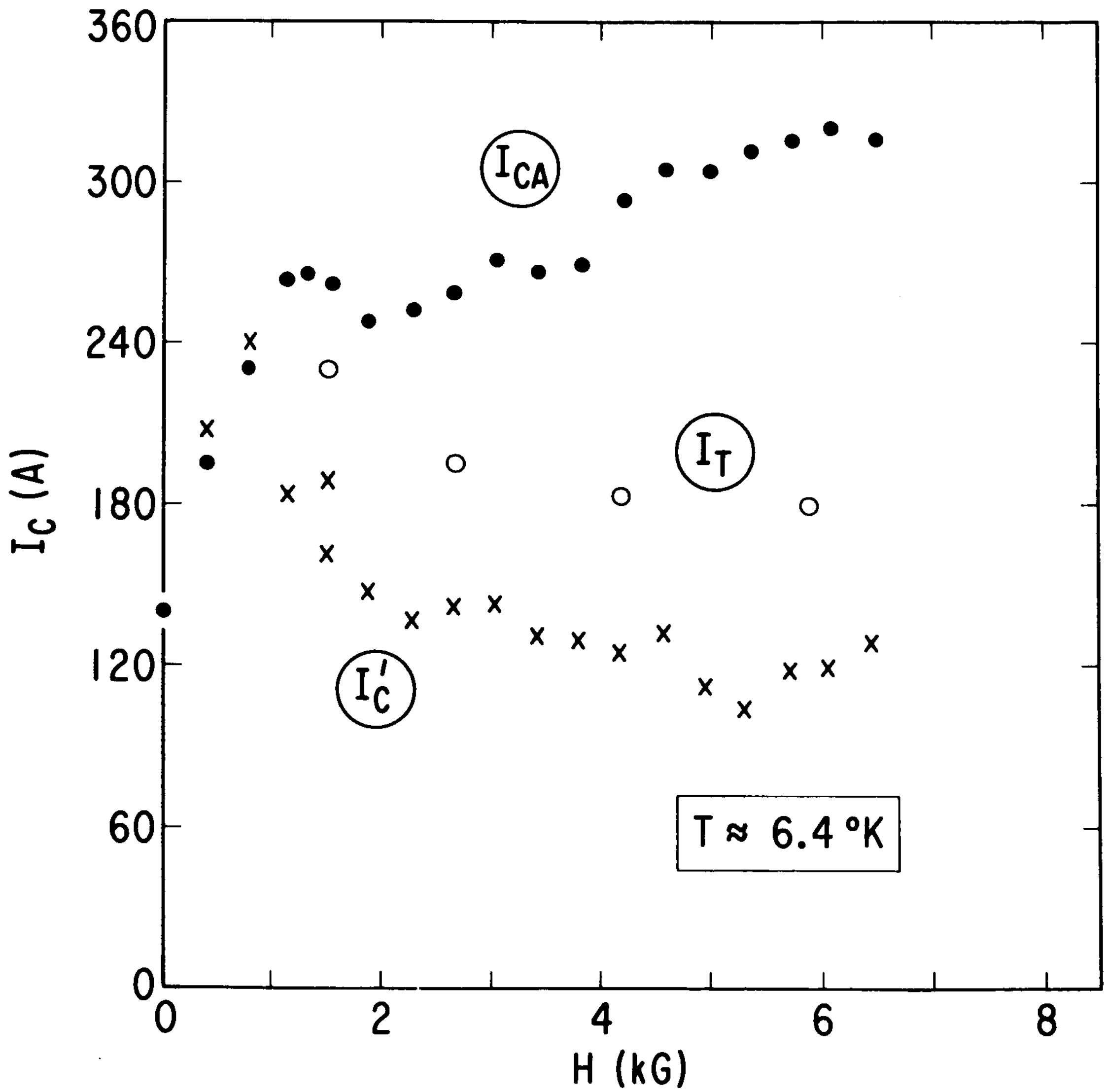

Figure 2

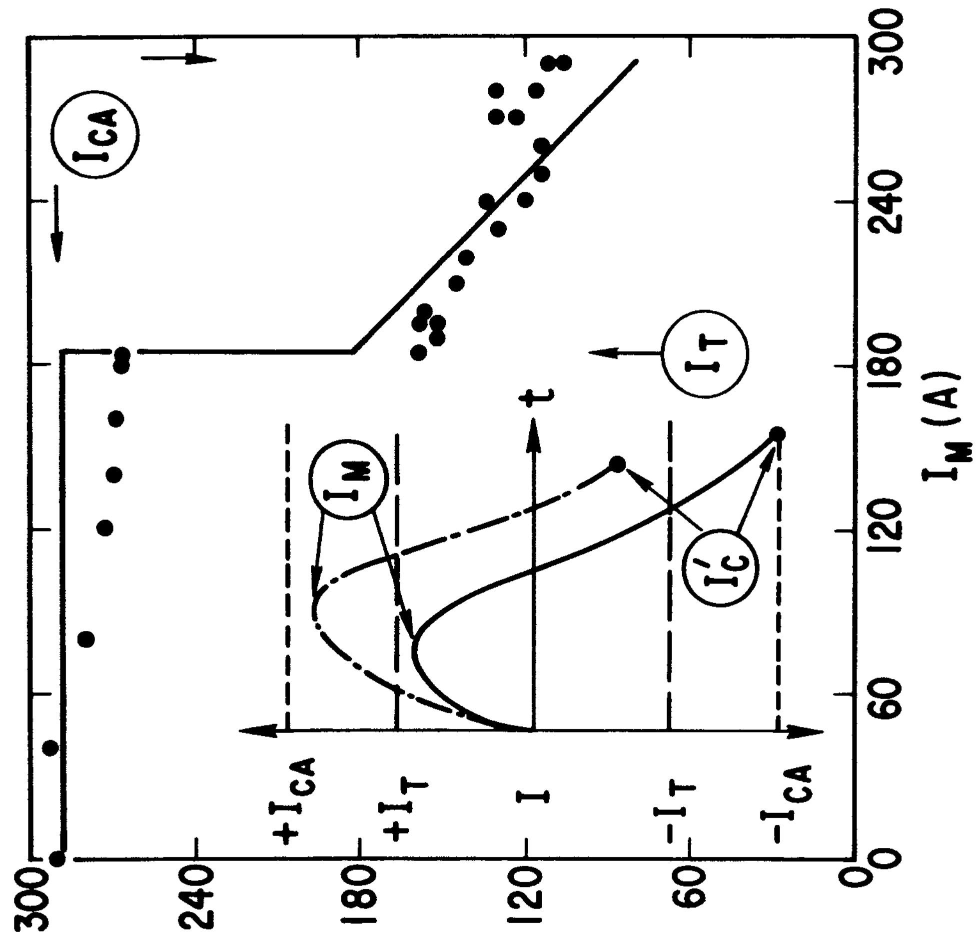

Figure 3

the threshold amplitude I_T required for the abrupt onset of degradation measures the longitudinal current carrying capacity of a surface sheath existing throughout the mixed state.

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Figure Captions

Figure 1. Remanent magnetization at H = 0 versus applied axial magnetic field when sample cools through T_c to 4.2^oK (x data points) and versus axial magnetic field applied to sample after cooling through T_c to 4.2^oK at H = 0 (· data points). Specimen is a 0.125 cm dia wire of NbTa.

Figure 2. Various critical currents versus axial magnetic field. I_{CA} is observed when I is increased continuously until quenching occurs. I_c' is observed after a cycle of the current in the opposite direction to a value near I_{CA}. I_T is determined from data such as presented in Figure 3. I_{CA}, I_T and I_c' are presented in the same quadrant for ease of comparison and economy of space.

Figure 3. The dependence of I_c' on I_M. I_M denotes the amplitude of the current during the cycle preceding the determination of I_c'. The polarity of I_M is opposite to that of I_c'. Solid curve shows theoretical behavior obtained from surface sheath model discussed in text. $T \approx 6.4^oK$, H = 4.2 kG, H//I.

Inset: Schematic display of the cycles of the current which lead to degradation of the critical current when the amplitude I_M exceeds a threshold value I_T.

B. Superconducting Thin Films as Microwave Circuit Elements

Grant AF-AFOSR-496-66, Joint Services Electronics Program

R. J. Pedersen, Z. A. Kaprielian

a) In the last progress report it was brought out that the very thin tin films measured (approximately 50 Å) did not behave precisely as predicted by the BCS theory. In addition to temperature dependent resistance and inductance as predicted, there was a capcitive effect which was discernable primarily from the reflection coefficient phase angle. A possible explanation was that the capacitive effects were introduced because the films were not uniform but consisted of small agglomerates separated by small gaps in which tunneling could take place. To test this hypothesis, masks were made to deposit contacts and a post type film on the substrate so that dc measurements could be made. These dc measurements could not be made on the uniform films because contact is made to the film on the four waveguide walls. Material was accumulated to make a solenoid external to the experimental chamber. Since a magnetic field would be a useful diagnostic tool especially if islands are close enough for Josephson tunneling, experiments were discontinued until the magnet could be wound.

b) A Josephson junction consists of two superconductors separated by a thin dielectric approximately 10 Å thick. The superconductors are thus in close enough proximity that a definite phase relationship exists between wave functions describing electron pairs on each side of the insulating barrier. These electron pair correlations give rise to dc and ac supercurrents which flow through the insulator and are known as the dc and ac

Josephson effects. The dc effect consists of a direct current flowing across the barrier without experiencing a voltage drop as would appear in a normal resistor. The maximum current is sensitive to magnetic field since the field affects the relative phase of the wave functions on each side of the junction. The ac effect consists of oscillating currents in the insulator with a frequency which is linearly related to the applied dc voltage. (483.6 MHz for each microvolt of applied voltage). These oscillating currents lead to radiation from the junction which may be detected.

Josephson junctions were fabricated by vacuum depositing a thin film of tin (about 2000 Å thick) on to a glass substrate and oxidizing this film in an oven at 110°C to form the thin insulating barrier. The junction was completed by then depositing a second tin film over the first. Measurements were made in liquid helium of a number of samples to determine the current-voltage relationship existing in the barrier. Most junctions showed peculiar properties indicative of short circuits. Several showed definite Josephson current and current spikes at finite voltages indicative of self-detection of ac Josephson currents. These spikes extended out to 500 GHz. Radiation was observed at 10 GHz when a film was current biased on a spike at 21 microvolts. The detected power level was about 10^{-12} watts and the direct current bias was 1 milliampere. This x-band signal was thus generated with a 2×10^{-8} watt input for a dc to ac conversion efficiency of 5×10^{-5}. This efficiency points up one of the problems associated with using a Josephson junction as a microwave circuit element. The junctions have such a low impedance that it is difficult to obtain a good match to normal waveguides.

During the next report period, it is planned to complete experiments to determine the nature of the capacitive effect in uniform thin superconducting films and thus to terminate that portion of the project. It is also planned to begin an analysis of Josephson junctions to analyze the mode structure and to seek ways to improve the match to external circuitry for greater power output.

II. APPLIED ELECTROMAGNETICS AND PLASMAS

2.1 PLASMAS

A. Wave Propagation in Bounded, Magnetized, Hot Plasmas

Grant AF-AFOSR-496-66, Joint Services Electronics Program

H. H. Kuehl, G. E. Stewart

Considerable theoretical and experimental interest has been generated recently in problems involving both thermal and instability produced radiation from hot plasmas. This radiation is of importance not only as an energy loss mechanism from hot plasmas but also as a means of indirectly observing various modes of propagation in plasmas. The electron cyclotron harmonic radiation has been measured by several investigators. In an early experiment by Landauer up to the 24th harmonic of the electron cyclotron frequency was observed in the radiation from a Philips Ion Gauge discharge[1]. Further measurements in such discharges have exhibited detectable radiation to as high as the 45th harmonic of the electron cyclotron frequency[2]. Similar radiation near harmonics of the electron cyclotron frequency has been observed from the positive columns of low pressure mercury and noble gas discharges in axial magnetic fields[3,4,5]. In each of these cases the observed radiation was several orders of magnitude greater than that predicted on the basis of a Maxwellian velocity distribution of electrons and cyclotron radiation from single particles.

There have been several explanations of this phenomenon ranging from the non-periodicity of the electron orbits which intercept the walls[6] to the excitation of the so-called Bernstein modes either through single particle interaction[7] or collective interaction with the usual beam-plasma instabilities[8]. A recent theoretical calculation has been made of the radiation by single particles in a warm, magnetized plasma of infinite extent[9], and some of the features of the experiments seem to be qualitatively explained in terms of this simple model. In particular, Lustig[10] has made simultaneous observations of the electron density and the power radiated in the positive column of a low pressure argon discharge, and he finds that this model can account for the approximate line position and width of one of the three lines observed at each of the harmonics of the cyclotron frequency. In addition to not explaining the other two peaks, the model fails to predict the strong dependence of the power radiated on electron density which was observed in the experiment.

Other recent studies have been directed towards understanding the coupling mechanism between the Bernstein modes and electromagnetic waves in plasmas of finite extent in order to explain the peaks in the noise spectrum. Careful studies of the absorption and radiation near each of the cyclotron resonances have revealed a structure which can be related to the finite cross-section of the plasma. The agreement with theory, at least for the structure around the second harmonic, in the location of these multiple peaks seems to confirm that they are indeed associated with the Bernstein modes[11, 12]. Another investigation has been able to establish semiquantitative agreement between theory and experiment for the resonant frequencies of these standing Bernstein modes in a cylindrical afterglow plasma[13]. The theoretical treatment was based on a slab model for the plasma and on a phase integral technique to solve a simplified form of the Boltzmann equation in both a strong field and weak field limit.

There have also been direct observations made of the propagation characteristics of the Bernstein modes across cylindrical plasmas[8, 14] and these add to the evidence that the noise peaks are due to radiation from the Bernstein modes, whatever may be their source of excitation.

In addition to the importance of Bernstein modes in determining the radiation from hot, magnetized plasmas, they also appear to be responsible for the "ringing" encountered in transmission from the Alouette satellite[15]. By examining the dispersion relation for a hot magnetized plasma, one investigator has been able to show that there are frequencies near the cyclotron harmonics where the group velocity of these waves goes to zero, and hence no energy is radiated from the satellite[16]. This leads to a large amount of energy stored in the near field, and a subsequent continuation of oscillation after the excitation is removed.

The Bernstein modes have the interesting property that, for propagation exactly normal to the magnetic field, they exhibit no collisionless damping; but for waves traveling obliquely with respect to the magnetic field there is damping which depends on the phase velocity in the direction of the magnetic field. However, one group of investigators has found from an approximate treatment of the problem that damping exists even for propagation normal to the magnetic field if there exists a zero-order electron density gradient in this direction[13]. Another interesting phenomenon is that the noise power radiated appears to peak in the vicinity of the 4th or 5th electron cyclotron harmonic. This has been explained in terms of trapping of the waves associated with the lower order harmonics inside the plasma by regions where the waves are cut off. These two phenomena lead to the conclusion that it will not be possible to obtain a full understanding of either the resonances in or the radiation from hot magnetized plasmas until more realistic problems involving the actual plasma and antenna geometries have been solved.

In order to better understand the coupling mechanisms between electromagnetic waves incident on a plasma and the Bernstein modes set up in the plasma, a correlated experimental and theoretical study has been undertaken. This investigation consists of both calculating and measuring the reflection and transmission coefficients of an electromagnetic wave incident normally on a hot, inhomogeneous electron plasma slab. For comparison, the problem of the hot, inhomogeneous electron plasma half space is also being solved. Theoretically, these problems possess the simplification that both the field quantities and the plasma vary in only one spatial dimension. Because of this simplification, in principle, this problem can be solved to almost any desired degree of sophistication, including the effects of a zero-order electric field, spatial inhomogeneity, and using the full set of Maxwell's equations instead of the usual electrostatic approximation. The wave equation for this special case becomes:

$$\frac{d^2\vec{E}_1}{dx^2} + k_0^2\vec{E}_1 - \frac{d^2E_{1x}}{dx^2}\,\hat{a}_x = -j\omega\mu_0\vec{i} \qquad (1)$$

where $\vec{i}$ is given by

$$\vec{i} = -e\int \vec{v}\, f_1(x,\vec{v},t)\, d^3v \qquad (2)$$

A similar equation can be written for the y-component of the electric field. The perturbation distribution function, f_1, can be evaluated by an integration along the zero order trajectories of the electrons, that is along paths which they follow in the absence of the R.F. fields. If the zero order electric field is such that the change in particle velocity is small in one gyration of the electric field, it is possible to use the guiding center approximation to compute the motion of the particles. The perturbed distribution function for an initially isothermal plasma with an isotropic distribution function can then be written as

$$f_1(x,\vec{v},t) = \frac{2e}{m}\int_0^\infty e^{-\nu s}\,\frac{df_0(x',v^2)}{d(v^2)}\left[\vec{v}\cdot\vec{E}(x,t)\right]' dx\ , \qquad (3)$$

where the prime indicates that the variables are to be evaluated at a time $t' = t - s$ on the zero order trajectory of the particle as it drifts perpendicular to both $\vec{E}_0$ and $\vec{B}_0$ and gyrates. By expanding the product of the electric field $\vec{E}_1$ and the unperturbed distribution function (assumed known) in Taylor series around x, it is possible to obtain an expression for the current at a given point in terms of an infinite series of derivatives of the R. F. electric field at that point. One must then find a solution which is consistent with the previously given wave equation, considering as many terms in the above expansion as are practical. The number of terms which must be included in the expansion correspond to the highest harmonic considered. This expansion yields the following coupled ordinary differential equations suitable for describing wave propagation below the second harmonic of the cyclotron frequency,

$$\frac{3}{2}\,\frac{v_{th}^2}{(\omega^2-\omega_c^2)(\omega^2-4\omega_c^2)}\,\frac{d^2}{dx^2}(\omega_p^2E_x) + \frac{\omega^2-(\omega_p^2+\omega_o^2)}{\omega_p^2(\omega^2-\omega_c^2)}(\omega_p^2E_x) \tag{4}$$

$$+\,j\,\frac{3\,v_{th}^2\,\omega_c}{\omega(\omega^2-\omega_c^2)(\omega^2-4\omega_c^2)}\,\frac{d^2(\omega_p^2E_y)}{dx^2} - j\frac{\omega_c}{\omega}\,\frac{(\omega_p^2E_y)}{(\omega^2-\omega_c^2)} = 0$$

$$\frac{v_{th}^2(\omega^2+\omega_o^2)}{2\omega^2(\omega^2-\omega_c^2)(\omega^2-4\omega_c^2)}\,\frac{d^2}{dx^2}(\omega_p^2E_y) + \frac{1}{k_o^2}\,\frac{d^2E_y}{dx^2} + \frac{\omega^2-(\omega_p^2+\omega_c^2)}{\omega_p^2(\omega^2-\omega_c^2)}(\omega_p^2E_y) \tag{5}$$

$$-\,j\,\frac{3\,v_{th}^2\,\omega_c}{\omega(\omega^2-\omega_c^2)(\omega^2-4\omega_c^2)}\,\frac{d^2}{dx^2}(\omega_p^2E_x) + j\,\frac{\omega_c}{\omega(\omega^2-\omega_c^2)}(\omega_p^2E_x) = 0.$$

For the case of a homogeneous plasma (ω_p^2 = constant) these equa tions can be combined, and exponential solutions assumed, yielding a fourth order determmental equation for the wave number. The solutions to this equation have been computed and compared with those calculated by Dnestrovskii and Kostomarov[17] using the exact solutions for the homogeneous plasma. The results are in good agreement in the regime where $k \; R_L < 1$.

The problem of a wave incident on an inhomogeneous, hot, magnetized plasma half space has been solved using the solutions of Eqs. 4 and 5, and a computer program for determining the reflection and transmission coefficients has been written. The problems of "mode conversion" and "tunneling"[15] at inhomogeneities are now being investigated using this formalism.

Experimentally, a device for generating a magnetized afterglow plasma in the form of a slab has been built and is ready to test. The gas is initially ionized by means of a several kilovolt pulse of microsecond duration applied between an anode and a segmented cathode. Transmission and reflection measurements will be made using a pair of focused X-band microwave horns.

This theoretical treatment and subsequent experimental verification should allow quantitative determination of the coupling between normally incident electromagnetic waves and the Bernstein modes (extraordinary waves). A comparison can also be made between the power radiated on the basis of Kirchhoff's law for this slab using the measured absorption coefficient and the actual radiated power since the electrons in the afterglow plasma should be close to equilibrium. This experiment can be carried out relatively easily using the Helmholtz coils and vacuum pumps that already exist in the steady-state hot-plasma facility which has been described in previous semiannual reports.

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2.2 RADIATION AND DIFFRACTION OF ELECTROMAGNETIC WAVES

A. Perturbation Method in the Diffraction of Electromagnetic Waves by Arbitrarily Shaped Penetrable Obstacles

Contract N123(60330)52008A, Naval Ordnance Test Station

C. Yeh

Exact solutions of boundary-value problems in the theory of electromagnetic wave diffraction are available only for certain specific bodies of relatively simple shape[1]. For example, the available exact solutions for cylindrical bodies without sharp edges are limited to those with circular, elliptic or parabolic cross-sections. The diffraction of waves by a conducting or dielectric sphere, by dielectric coated spheres and by a perfectly conducting disk are the few three dimensional problems that have been solved rigorously. The need for approximate methods to treat the more general cases of diffraction from arbitrarily shaped obstacles is quite apparent. The variational principles[2] provide a very powerful tool in obtaining approximate expression for the scattering cross section; but it is not possible to derive from the variational principle a description of the electromagnetic fields. Furthermore, the success of the variational approach depends to a great extent on the trial function. At low frequencies, the Rayleigh[3] method is very successful. However, the solutions of Laplace's equation are still required. At very high frequencies, the treatment of diffraction problems by geometric and physical optics techniques developed by Fock[4] and Keller[5] is very successful. An approximate or perturbation method in the medium frequency range still remains to be found.

In the present work the boundary perturbation technique which is based on a Taylor expansion of the boundary conditions at the perturbed boundary will be extended to consider the problem of the diffraction of waves by a dielectric object with perturbed boundary. Since this approach attacks the complete boundary-value problems, the perturbation solution for the field components is valid for the near zone as well as for the far zone and is valid for all frequencies. In a way of illustration, the problem of the diffraction of electromagnetic waves by a dielectric cylinder with perturbed boundary was treated. A specific example on the scattering of plane waves by a dielectric elliptic cylinder with small eccentricity was given. Results were compared with those obtained from the exact solution. The more involved case of the diffraction by a dielectric sphere with perturbed boundary can be solved in a similar manner[6].

Results of this investigation have been published in the Journal of Mathematical Physics[7].

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B. Reflection and Transmission of Electromagnetic Waves by a Moving Dielectric Medium

Grant AF-AFOSR-496-66, Joint Services Electronics Program

Contract N123(60330)52008A, Naval Ordnance Test Station

C. Yeh

The purpose of this investigation is to obtain the solution to the basic problem of the reflection and transmission of plane waves by a uniformly moving dielectric half space. The reflection and transmission coefficients are determined. Two cases of the movement are considered: (a) the dielectric medium moves parallel to the interface. (b) the dielectric medium moves perpendicular to the interface. Various interesting features concerning the variation of the reflection and transmission coefficients, angles of reflection and transmission, and the frequencies of the reflected and transmitted wave, as a function of the velocity of the moving medium, are obtained.

Results and detailed discussions have appeared in the Journal of Applied Physics[1].

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C. Radial Prolate Spheroidal Wave Functions

Contract N123(60330)52008A, Naval Ordnance Test Station

C. Yeh, C. Chang

Although prolate spheroidal functions have appeared in such diverse fields as: the electromagnetic and acoustic scattering or radiation, the two center problem is quantum mechanics, the viscous fluid flow, stochastic processes and recently in such fields as the uncertainty principle, the determination of laser modes, and radiation from dielectric coated spheroidal antenna, these functions have always been used with reluctance. This is because these prolate spheroidal functions had not been tabulated extensively enough, and in order to find numerical values, very laborious computations had to be carried out. Furthermore, the slow convergence properties of various expansions of prolate spheroidal functions leaves much to be desired.

The purpose of the present investigation is to make an extensive and systematic tabulation of prolate radial functions. It is intended that the results be used for the numerical calculation of the diffraction of waves by a prolate spheroid.

2.3 MILIMETER WAVES

A. Millimeter-Wave Radiometry for Radio Astronomy

Contract 951 424, Jet Propulsion Laboratory

W. V. T. Rusch, S. Slobin, C. Stelzried

a) Lunar Eclipse Observations:

Radiometric observations of a total lunar eclipse were carried out at a wavelength of 3.3 mm from a site in Palm Springs, California, during the night 18-19 December 1964 (Pacific Standard Time). Eleven drift curves (with analog and digital output) were made before the end of the umbral phase and a total of 71 drift curves were made during the night.

Instrumentation. The observations were made with a five-foot search-light reflector modified to operate as a Cassegrainian-fed antenna. The 3.3 mm Dicke-type superheterodyne radiometer[1] employed a switchable four-port circulator (TRG Model #E161) ahead of the mixer to switch between the antenna and a reference load. A ferrite isolator (TRG Model #E110), with 0.25 db insertion loss and 23 db isolation, was used to reduce rf mismatch errors. Silicon diodes were used in the Raytheon balanced mixer. The waveguide transmission loss between the antenna and the mixer was 2.2 db (including 1.1 db loss in the circulator). The IF passband extended from 22 mc to 75 mc. The output jitter measured at the output of the manual waveguide switch was $5^{o}K$, rms, with a ten-second post-detection time constant.

Antenna gain, pattern, and tracking characteristics were measured on two antenna ranges, at distances of $1.5\ D^2/\lambda$ and $5D^2/\lambda$. The measured half-power beamwidth was 10.5 minutes of arc, and the equivalent aperture efficiency of the antenna, including losses in the manual waveguide switch, feedhorn, and mirror coating, was 0.24.

The moon was tracked using a 40-power sighting telescope alligned with the antenna main beam. Previous tests indicated that reference points on the moon could be tracked with a maximum error of one minute of arc. Convolution of the antenna pattern with isothermal maps of full moon[2] indicate a one-minute tracking error contribute an error of 1% or less in the radiometer output, depending upon the direction of the error.

The gas tube was calibrated by repeatedly switching the manual waveguide switch between a hot load (at a temperature of 50^{o}C), a second hot load (150^{o}C), and the gas tube.

Results. The eclipse observations, as carried out with the 10.5-minute search-light antenna beam, indicate no significant temperature difference between the data taken during the final umbral phase of the eclipse and the data taken 2-5 hours after the end of totality. The probable error of the measurement was estimated to be 2% of the final moon temperature.

Conversion factors obtained from calibration of the gas-tube equivalent excess noise temperature, the antenna gain, and the antenna radiation pattern yield the equivalent black-body disc temperature of the moon corresponding to $T_M'/T_{GT} = 1.138$ was 288^{o}K. Thus the estimated probable error of the measurement was $6^{o}K$.

Systematic error in absolute thermal calibration of the radiometer (estimated to be 7-9%) would affect the absolute level of the equivalent disc temperature but not the percentage change during the eclipse.

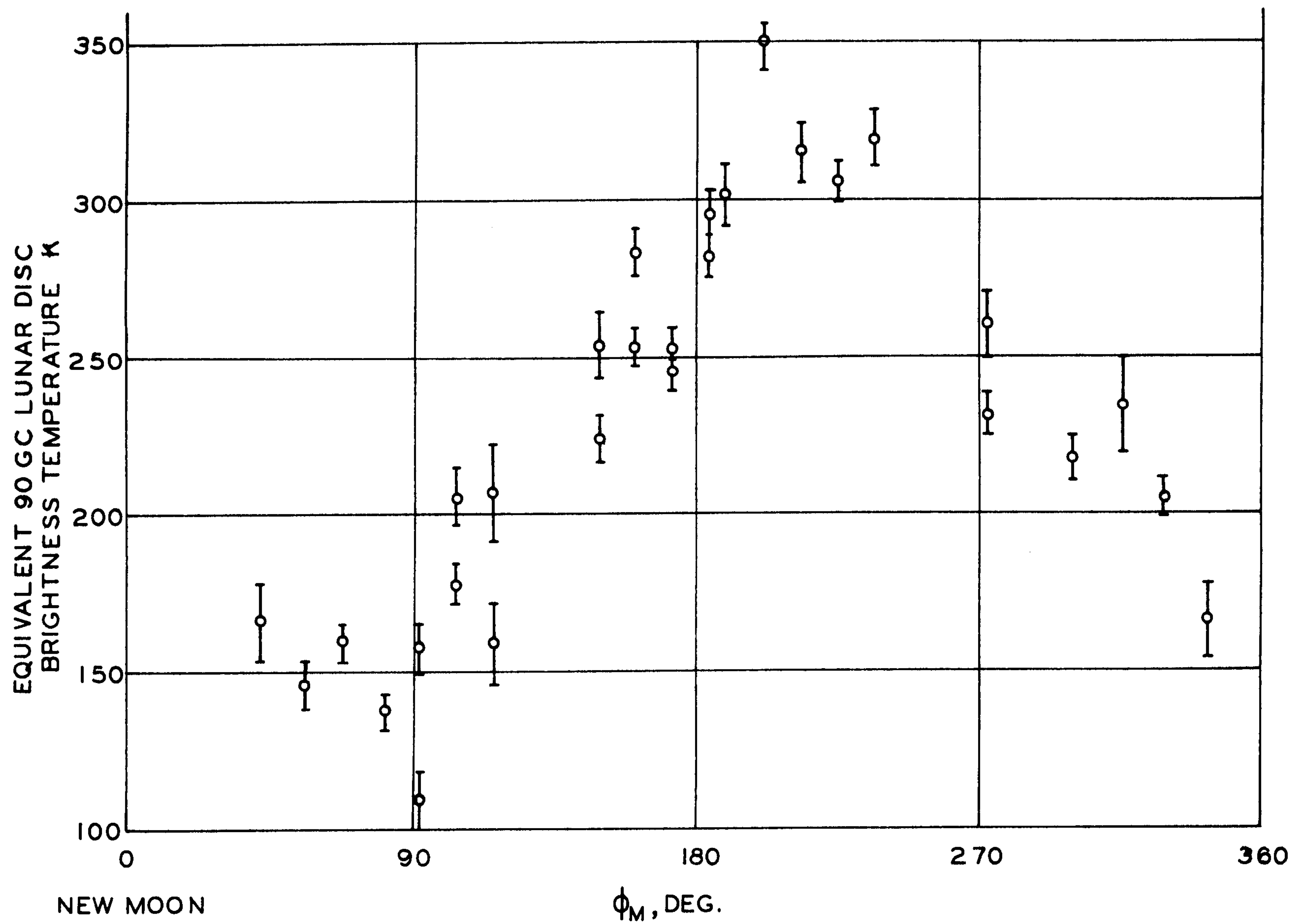
b) Lunation Observations:

Radiometric observations of the sub-earth point on the moon were carried out daily during the period from 12 August 1965 to 12 September 1965. This period of time extended from a lunar phase of 190 deg, lunation 527, to a phase of 209 deg, lunation 528. These lunation observations, which yielded data of considerable scientific interest, also provided an excellent test for the long-term stability of the radiometer. Thermal calibration of the gas tube before and after each series of observations produced information on the long-term gas tube output. Another useful by-product of the observations was the data on atmospheric absorption.

The results of the observations are plotted in Figure 1 in terms of T_{MD}, the equivalent blackbody disc temperature of the moon as a function of lunar phase. In many cases the probable error flags, computed from the statistical scatter of the observations, do not overlap to produce a smooth curve. This effect is atributed to temporal variations in the atmospheric absorption during a series of observations.

c) Solar Observations:

Observations of the sun were carried out from 8 February 1966 to 19 February 1966 to measure the solar radiometric brightness at 3.3 mm.



Equivalent Lunar Blackbody Disc Temperature vs. Phase

Figure 1

The results of these observations are presently being evaluated.

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III. INFORMATION SCIENCES

3.1 CONTROL SYSTEMS

A. General Theory of Optimization

Grant AF-AFOSR-1029-66, Air Force Office of Scientific Research

L. W. Neustadt

In the past six months the general theory of optimization processes previously developed has been broadened so as to apply to an even wider class of problems. Those include mathematical programming problems in abstract spaces (for which generalized Kuhn-Tucker conditions have been obtained), and optimal control problems with equality constraints in infinite-dimensional spaces. These results, as well as those developed earlier, have been written up in the form of three papers to be submitted for publication in professional journals.

B. Finite State Attitude Control

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

C. R. Walli, I. S. Reed, R. B. McGhee

The first phase of research on this project has been completed and published as an interim technical report[1]. This report includes phase plane and describing function analysis as well as simulation results which demonstrate the feasibility of finite state attitude control[2]. Analytic studies now under way are aimed at adapting the Lyapunoff theory of stability to finite state systems with memory in order to provide a more fundamental basis for the study of finite state attitude control. Another analytic investigation has been initiated with the objective of determining fuel-optimal finite state control laws. A computational algorithm for obtaining such laws by solving a sequence of dynamic programming problems is being developed.

The design and construction of a low friction single axis test stand for the experimental verification of single flip-flop attitude control has been completed. An air bottle and gas jets are now being installed. It is expected that preliminary testing of this system will begin in the very near future.

C. Optimal Control in Rendezvous, Pursuit and Similar Two Body Problems

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

J. Murrin, N. E. Nahi

We are concerned with systems with more than one set of controls. Each control set is used by a different individual who attempts to choose his control so as to extremalize (maximize or minimize) his measure of system performance. Of particular interest are those cases where the various performance measures are different or where the abilities of the various control sets to affect system motion differ.

A set of control strategies is optimal by definition if varying any one (and only one) causes its user to suffer. In the last six months the nature of such optimal sets of strategies has been investigated. In particular, we have looked at their existence, conditions for optimality, the stability of the system motion when such control strategies are used, and situations which justify use of such a definition of optimality.

It appears that the optimal control strategies tend to be appreciably more complicated than their counterparts in control problems with only a single control set. For example, we have yet to find a problem for which the optimal strategies can be stated in closed form. The current effort, therefore, is to develop one or more reasonably useful methods of synthesizing the optimal strategies. Direct computational schemes, successive approximations and other methods will be looked at in the future.

D. Optimum Control of Distributed Parameter Systems

Contract AF-AFOSR-496-66, Joint Services Electronics Program

Contract AF-AFOSR-1026-66, Air Force Office of Scientific Research

P. K. C. Wang

The problem of determining optimum control laws for distributed parameter dynamical systems describable by partial differential equations, integral equations or functional differential equations is generally very difficult. In most cases, approximation in one form or another is imperative in the implementation and/or the determination of the control laws. Moreover, the complete stability of the resulting controlled system is not always guaranteed. From the application standpoint, it is very desirable to have methods which will treat the optimization, approximation and stability problem in a unified manner.

During the period covered by this report, a method due to Bass[1] for designing lumped parameter control systems has been extended to distributed parameter systems. The method leads to feedback control systems which are completely stable and have improved speeds of response over those of the corresponding uncontrolled systems. Specific results have been obtained for systems describable by parabolic and hyperbolic partial differential equations with controls introduced at the boundary and in the interior of the system's spatial domains. Nume rical examples have been worked out to substantiate the validity of the theoretical results. This work has been reported in Ref. 2.

References

1. Bass, R. W., "Discussion of a paper by Letov", Proc. Heidelberg Conf. on Auto. Control, (Regelungstechnik: Moderne Theorien und ihre Verwendbarkeit, R. Oldenbourg, Munich), 1957, p. 209.

2. Wang, P. K. C., "On the feedback control of distributed parameter systems", USCEE Report No. 165, March 1966.

E. Stability Analysis of Distributed Parameter Control Systems

Contract AF-AFOSR-496-66, Joint Services Electronics Program

Contract AF-AFOSR-1026-66, Air Force Office of Scientific Research

P. K. C. Wang

The main objective here is to apply and to extend various qualitative and quantitative methods in partial differential equations for the stability analysis of distributed parameter control systems.

Preliminary results of this study have been reported in a paper which appeared in the IEEE Transactions on Automatic Control[1]. Recently, the stability problem associated with a particular class of mixed distributed and lumped parameter feedback control system has been studied. These systems are in the form of a parabolic partial differential equation coupled with a nonlinear ordinary differential equation. Sufficient conditions for stability have been derived by using a weak maximum principle for parabolic equations. The results have been described in a recent report[2].

References

1. P. K. C. Wang, "Asymptotic Stability of Distributed Parameter Systems with Feedback Controls", IEEE Trans. on Auto. Control, Vol. AC-11, No. 1, Jan. 1966, pp. 46-54.

2. P. K. C. Wang, "On the Stability of Equilibrium of a Mixed Distributed and Lumped Parameter Control System", USCEE Report 157, January 1966.

F. Stability Problems in Dynamical Systems with Random Parameters

Contract AF-AFOSR-496-66, Joint Services Electronics Program

P. K. C. Wang

The results reported earlier pertaining to the almost sure stability of linear distributed parameter system and linear time-lag systems with stochastic parameters have been published[1,2]. Attempts have been made in sharpening the stability conditions obtained earlier and also in deriving sufficient conditions for almost sure instability. The latter task was found to be quite difficult. Further work in this direction will be continued.

References

1. P. K. C. Wang, "On the almost sure stability of linear distributed parameter dynamical systems with stochastic parameters", J. of Applied Mechanics, Vol. 33, Series E, No. 1, March 1966, pp. 182-186.
2. P. K. C. Wang, "On the Almost Sure Stability of Linear Time-Lag Systems with Stochastic Parameters", International Journal of Control, Vol. III, No. 1, 1966.

G. Studies on the Intrinsic Properties of Dynamical Systems

Contract AF-AFOSR-496-66, Joint Services Electronics Program

P. K. C. Wang

One of the basic problems in control theory is to identify those intrinsic properties of dynamical systems which are of importance to control. Attempts have been made to clasify various intrinsic properties such as controllability, observability, invariance, sensitivity, etc., under a common framework, and to establish relations among these properties. Preliminary efforts have been devoted to formulating the problem in the framework of an abstract dynamical system. Future work will be specialized first to linear dynamical systems and then to nonlinear dynamical systems.

H. Optimum Propulsion of Flexible Hydrofoils

Contract AF-AFOSR-496-66, Joint Services Electronics Program

P. K. C. Wang

Preliminary results of this work were described in the previous progress report and USCEE Report No. 143. During the period covered by this report, the optimum propulsion problem for an oscillating flexible foil has been considered. It was shown that the optimization problem is reducible to a nonlinear programming problem. Attempts will be made in obtaining numerical solutions to this problem.

I. Inverse Problems in Adaptive Controls

Contract AF-AFOSR-1029-66, Air Force Office of Scientific Research

D. D. Sworder

There exists a rather distinct dichotomy in the published work on adaptive control. On the one hand, there has been an effort to develop analytical tools which will permit the engineer to synthesize an optimal or best adaptive controller. Coincident with this work, but seemingly independent of it, numerous "learning systems" have been proposed which hold promise of application in actual design problems. The basis for this division must rest, at least in part, on the fact that the criterion functionals amenable to analytic treatment do not usually provide an adequate description of the design problem. In Reference 1 the inverse problem is studied; that is, when does the engineer behave as if he were designing a controller to minimize the expected value of some criterion functional? It is shown in this reference that under relatively weak conditions there does exist a performance measure of a specific form which agrees with the engineer's behavior pattern.

Reference:

1. D. D. Sworder, "An Inverse Problem in Discrete-Time Adaptive Control", USCEE Report 164.

J. Control of Discrete-Time Stochastic Systems

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

D. D. Sworder

The optimal control of a linear stochastic system with a quadratic performance measure was investigated in the presence of a bound on the input. A technique formally similar to the maximum principle was employed to derive a recurrence formula for the optimal control. The bound on the control complicates the synthesis for it is clear that the optimal control is not a linear function of state as it is in the unconstrained problem. It was shown that the main effect of the bound was to modify the equation for a variable anlogous to the adjoint variable in the maximum principle formalism.

Reference

1. D. D. Sworder, "Control of a Linear Discrete-Time Stochastic System With a Bounded Input", USCEE Report 150.

K. Optimal Feedback Control

Contract AF-AFOSR-1029-66, Air Force Office of Scientific Research

D. D. Sworder

In Reference 1 a study was made of the optimal feedback control of a class of stochastic systems. Because of its utility in deterministic problems, it is natural that the maximum principle should be extended in the direction of stochastic systems. In fact it has been shown by other investigators that the maximum principle finds direct application in determining the optimal open-loop control for certain stochastic problems. For stochastic systems, however, open-loop controls are not equivalent to closed loop controls in the same sense they were in the deterministic situation, for now knowledge of the initial state and past control action does not implicitly specify the current state. It is shown in Reference 1 that the differential equation for the co-state process must be modified to obtain the optimal feedback control.

Reference

1. D. D. Sworder, "Optimal Feedback Control of a Class of Stochastic Systems", USCEE Report 154.

L. Dynamic Programming

Grant AF-AFOSR-496-66, Joint Services Electronics Program

R. Bellman

Effort during this report period was devoted to extending the theory and applicability of dynamic programming. Studies have also begun in the solution of biomedical problems using the dynamic programming formalism. New numerical techniques for digital computer solutions of these problems have been developed. Papers written include:

1. "Functional Equations in the Theory of Dynamic Programming--XII: Stability Considerations," to appear in J. Math. Anal. Appl.

2. "New Methods for the Solution of Partial Differential Equations," with R. Kalaba, for Proceedings of Symposium at University of Delaware.

3. "Dynamic Programming: A Mathematical Theory of Decisionmaking," to appear in SCIENCE.

4. "Relative Invariants, Closure, and Stochastic Differential Equations," with J.M. Richardson, to appear in J. Math. Anal. Appl.

5. "Invariant Imbedding and Radiative Transfer in Spherical Shells," with H. Kagiwada and R. Kalaba, to appear in J. Computer Physics.

6. "The Roles of a Mathematician in Applied Mathematics," for Proceedings of Symposium at the University of Minnesota.

7. "Adaptive Processes and Intelligent Machines," for Proceedings of Symposium at University of California, Berkeley.

8. "The Body Politic, Automation, and Bioengineering," to appear in The NATION.

9. "A Technique for the Analysis of a Broad Class of Biological Systems," with R. Roth, for Proceedings of Bionics Symposium, Dayton, Ohio.

10. "Dynamic Programming and Modern Control Theory," for International Congress of Mathematicians, Moscow.

Books in Progress:

1. Introduction to Modern Control Theory
2. Modern Elementary Differential Equations
3. Cybernetics, with I. S. Bengelsdorf
4. Closure Techniques in Mathematical Physics, with J.M.Richardson
5. Algorithmic Processes, Combinatorics, and Computers, with K. L. Cooke and J. Lockett

M. Optimum Guidance of Space Vehicles

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

N. E. Nahi, L. A. Wheeler

Transfer of a space vehicle from a set of initial conditions to a desired set of final values, while minimizing the time of transfer or fuel consumption or minimizing the terminal miss distance, is one of the important problems concerning the design of a space vehicle system. A procedure has been developed which takes the advantage of the assumption that the space vehicle can be represented by a set of linear, time varying differential equations to yield a particularly simple optimization technique. In effect the original problem is converted into a sequence of far simpler problems for which the solutions can readily be obtained. The advantage of this technique as compared to other known techniques (Maximum principle, Dynamic programming, Functional analysis) is its simplicity of mechanization and, this consequently, makes it possible to apply to space vehicle systems where the size, weight and reliability play extremely important roles.

Special purpose airborne computers built with current available components could be used to apply this technique on board. The technique may be used to determine the function of an optimum autopilot of the space vehicle. It may also be used to derive the optimal trajectory for the vehicle. The technique discussed above has been tested with realistic example problems and found to perform successfully.

A technique was developed for solving the continuous terminal control problem. This technique involves solving a sequence of discrete terminal control problems. It was proven that the sequence converges and yields the optimum continuous control. In this approach any technique could be used to solve the discrete terminal control problem. Furthermore, a technique was developed for solving the discrete terminal control problem in a very efficient way. It is the technique which was used to solve the discrete terminal problem in the solution of the continuous case. This technique utilizes the geometric properties of the discrete reachable set and the characteristics of the optimum control. This formulation does not require the manipulation of any matrices of higher order than the differential equations describing the system. A report covering this work is in preparation.

In addition the closed loop terminal control problem was studied in this period. The techniques which have been developed for solving the open loop terminal control have the property that the problem solution time is directly related to the distance in state space from the current state to the optimum state. This property can be utilized in the solution of the closed loop problem.

The closed loop problem will be solved by resolving the open loop problem at each sampling time based on the current miss vector. The proposed technique has an important advantage over the application of other open loop techniques in this fashion. This advantage is that the amount of computation time needed at each step is directly related to the difference between the predicted state at this step and the actual measured state. Therefore the amount of computation time needed to recompute the optimum control is a direct function of the accuracy of the representation of the system by the set of differential equations used in the optimization procedure. This property plus the computational simplicity of the basic technique yields an effective approach to the on-board solution of the closed loop optimum terminal control problem for space vehicle applications.

3.2 INFORMATION PROCESSING AND SIGNAL DESIGN THEORY

A. Television Bandwidth Reduction Studies

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

W. K. Pratt, H. C. Andrews

A novel method of forming a difference signal which is proportional to the intensity subtraction of two time adjacent television frames has been developed. The difference signal provides a measure of the redundancy between frames, and is useful in the development of pattern recognition and television bandwidth reduction systems.

The difference in frames is generated optically using the properties of light intensity addition of positive and negative transparencies of time adjacent frames. Positive and negative slides of the same scene are projected onto a translucent screen from opposite but colinear positions perpendicular to the screen (see Figure 1). Detail cancellation to a neutral gray level theoretically occurs since if white is projected by one slide, black is projected by the other.

Using this artificial frame difference generation technique, various types of camera motions can be simulated. With identical scenes, panning in a horizontal or vertical direction can be achieved by horizontal or vertical displacement of one of the projectors from its colinear position. Camera roll can be achieved by rotating one of the projectors, and zooming obtained by placing one projector nearer to the screen than the other. For

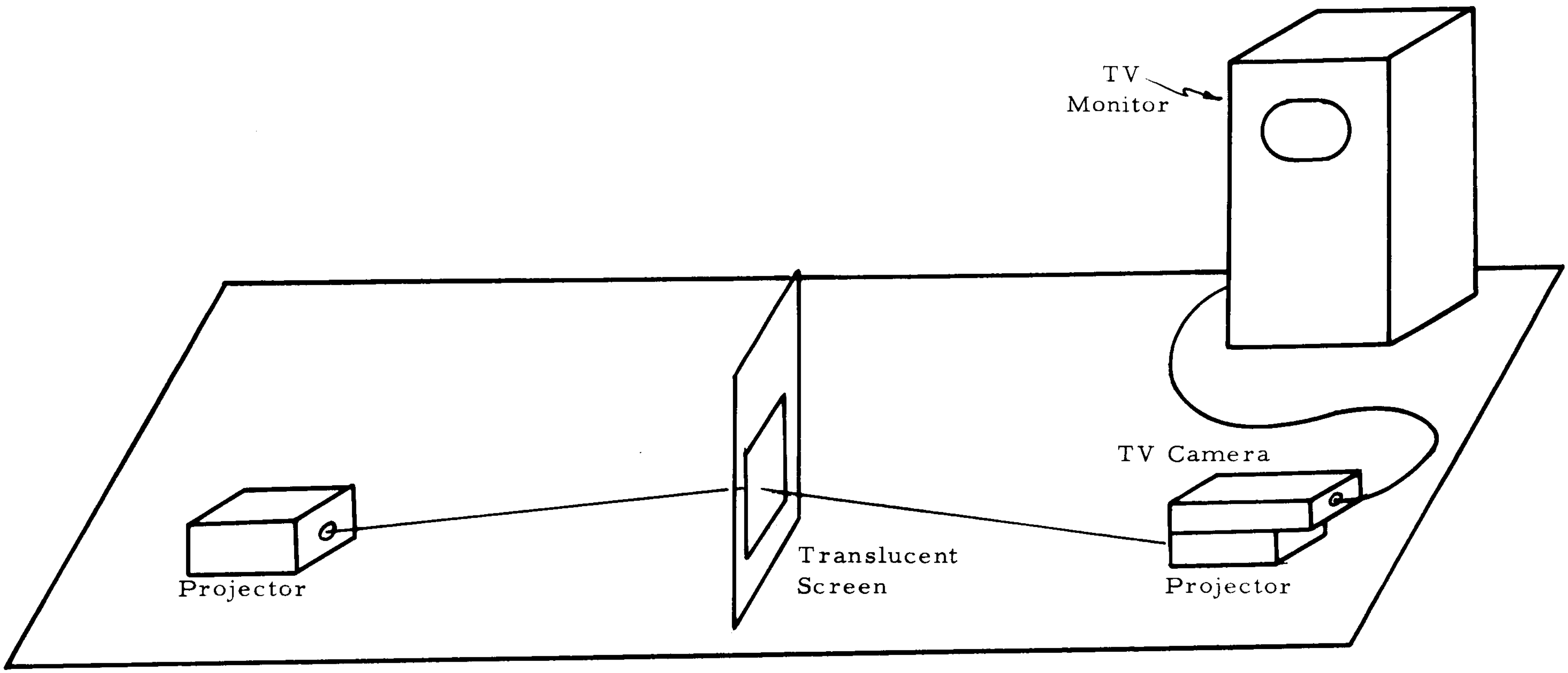

TV
Monitor
TV Camera
Projector
Translucent
Screen
Projector

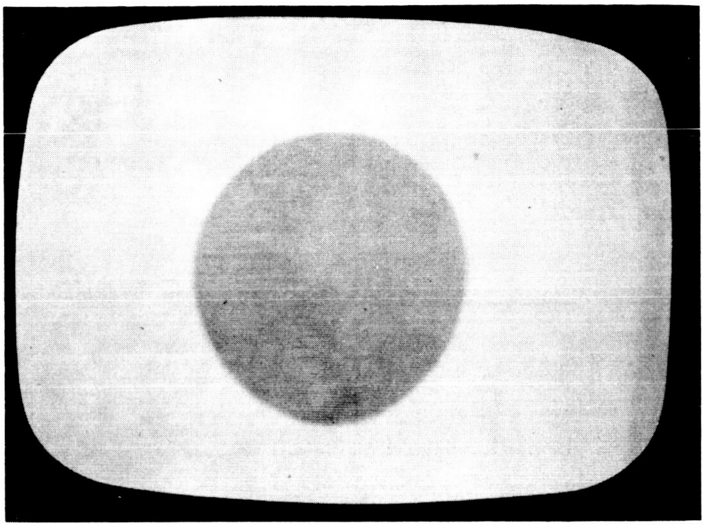
Figure 1

movement of objects within a scene, slides of time adjacent frames are used. In each of the examples sited, the image on the screen, when scanned with a TV camera, produces a frame difference video signal simulating the difference in time-adjacent frames.

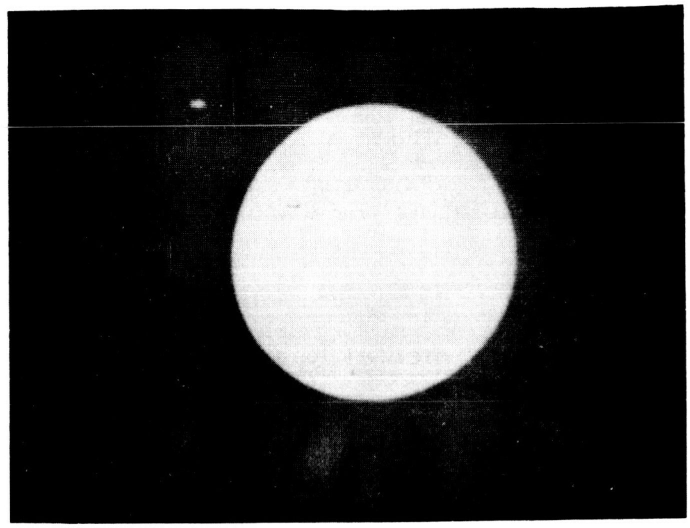
Experimental Results

Figures 2a and 2b show black ball positive and negative transparencies as viewed on the television monitor. The corresponding video signals of one TV line are shown in figures 2c and 2d. Figure 3a shows the detail cancellation when the positive and negative black ball transparencies are superimposed on the translucent screen. A simulation of motion (i. e. a ball moving across the field of view, or alternately, the camera panning a stationary ball) is illustrated by the difference frame of figure 3b. The video signals for these respective difference frames are shown in figures 3c and 3d.

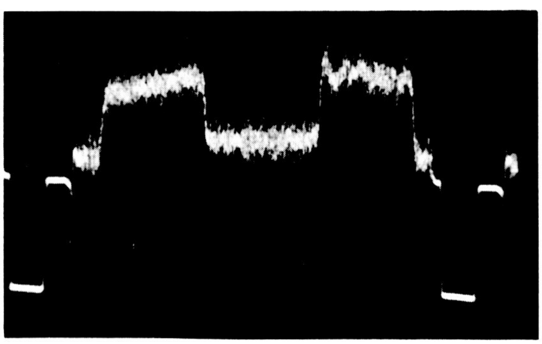
Figure 4a shows the positive transparency of a moon scene as viewed by the monitor. Detail cancellation in this scene is apparent from the difference frame of figure 4b.



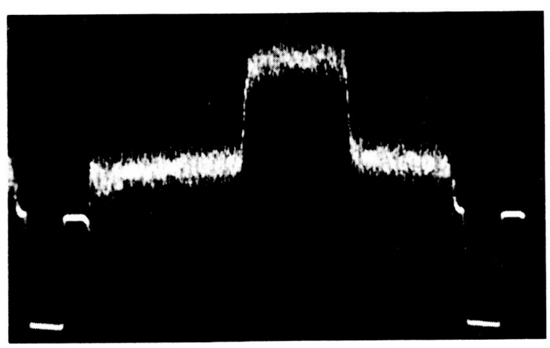
2a



2b

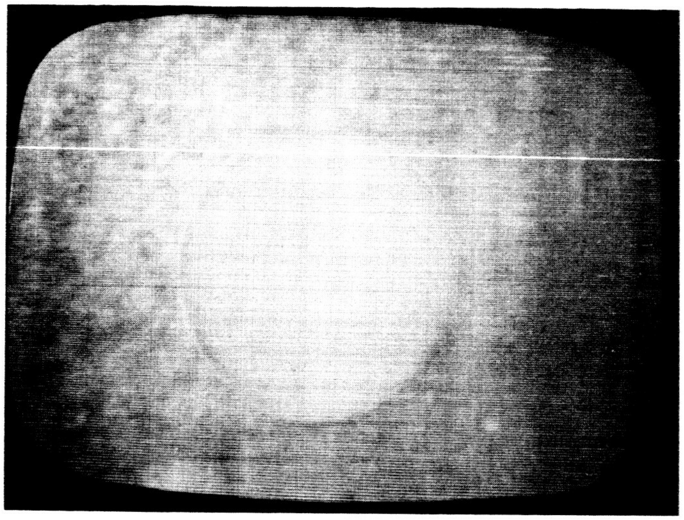


2c

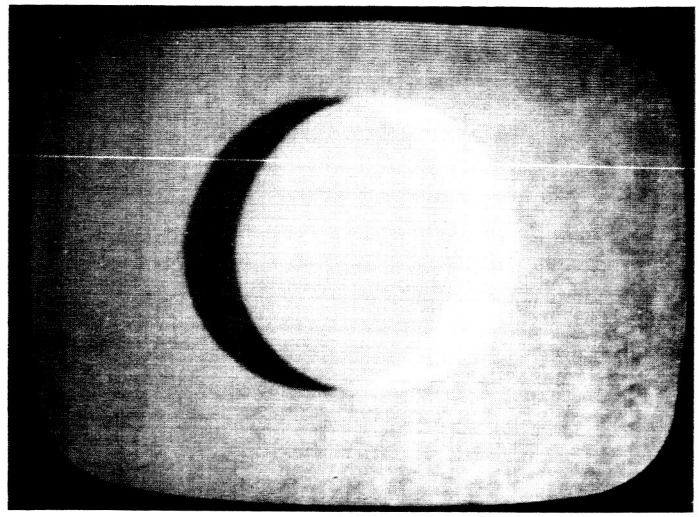


2d

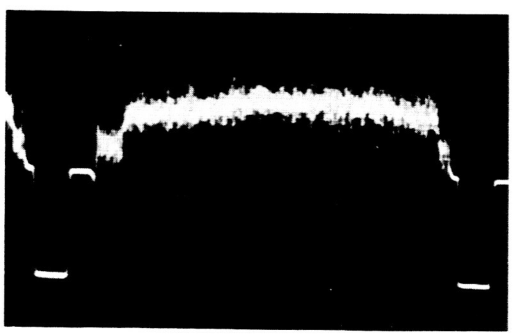
Figure 2



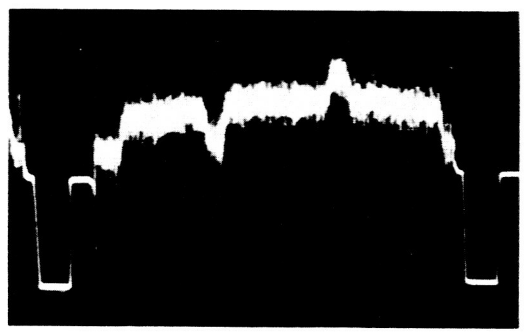
3a



3b

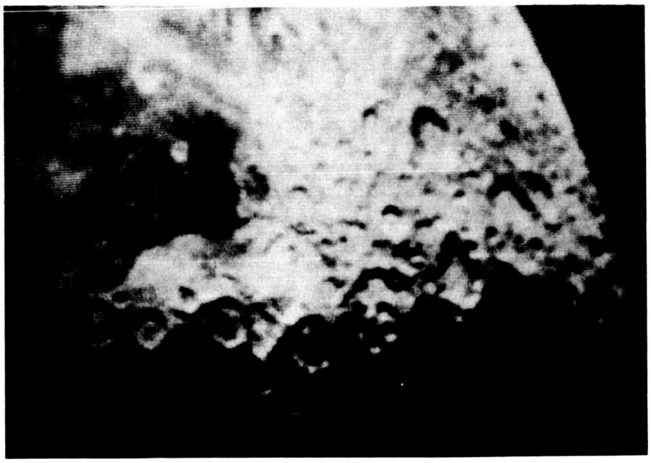


3c

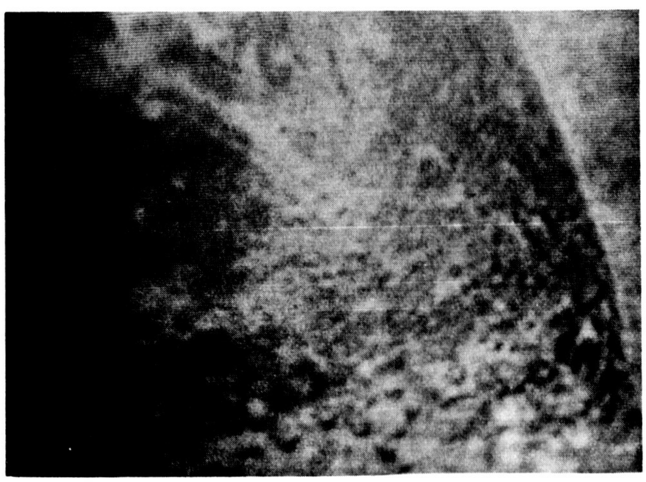


3d

Figure 3



4a



4b

Figure 4

B. Laser Modulation and Detection Study

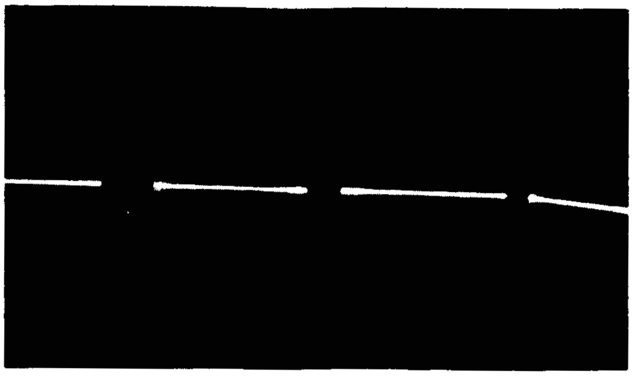
SSD

W. K. Pratt, R. Norton

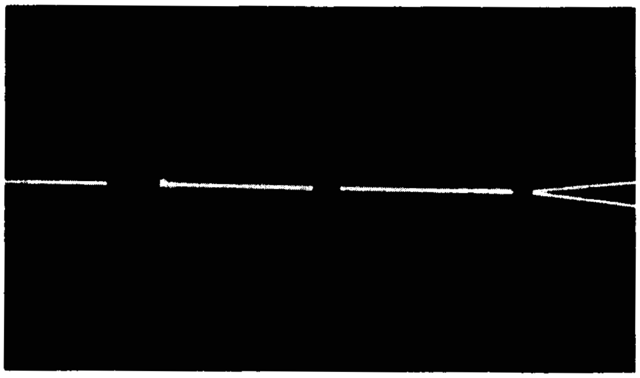
A laser polarization modulation communication system is under development. The system transmits bits of information as pulses of either right or left circular polarization. Utilization of all the energy emitted by the laser results in an improved signal-to-noise ratio at the receiver over an intensity modulated system. High rates of transmission of data are also possible.

The photograph in Figure 1 illustrates results of a test conducted using a quarter-wave plate to simulate the laser modulator. The receiver uses a quarter wave plate to convert the light of two circular polarizations to orthogonal polarizations and directs them to twin photomultiplier tubes. One hundred percent modulation results in the beam being directed into one of the photomultipliers while fifty percent modulation results in splitting the energy between the two.

In an associated analytical study, USCEE Report 149, SSD-TR-65-162, titled, "Binary Detection in an Optical Polarization Modulation Communication Channel", has been issued. The report derives the exact probability of detection error and compares the result to that obtained from a Gaussian approximation.



(a) Circular Polarization



(b) Elliptical Polarization

components left to right: modulator, quarter wave plate, Wollaston prism

FIGURE 1 - STATIC MODULATION AND DETECTION

C. A Unified Model for Laser Communications and Radar

SSD

S. Karp, R. M. Gagliardi

Communication engineers are hampered in their efforts to design laser systems by the lack of a consistent radiation and receiver model. While the simple detection of laser sources can be analyzed conveniently based upon a particle model for the radiation and a particle-counting model for a receiver, the advantages of possible optical heterodyning can only be described from a wave representation of the radiation. The deficiency of the analysis procedures further impedes the consideration of channel effects such as dispersion, etc. In this research effort we have attempted to develop a unified system model applicable to any laser system. The approach has been to use statistical wave pockets to describe the impinging radiation which, with a slight modification, can describe heterodyning behavior. The model has been convenient for handling the effects of statistical transmission media, such as rough terrain, sea, and rough scattering surfaces, and their effect on detected signals is quite easily determined. The results predicted have been highly correlated with known measured data. Some interesting extensions of laser detection theory, reported in the literature, have also been achieved.

D. Signal Design Theory

Grant AF-AFOSR-496-66, Joint Services Electronics Program

R. A. Scholtz, C. L. Weber, L. Welch

A significant simplification of the detection probability for noncoherent synchronous channels has been obtained. The new representation is

$$P_D(\lambda^2; \Gamma) = \frac{1}{n} e^{-\frac{1}{2}\lambda^2} E\left[I_0(\lambda \max_i \rho_i) \right]$$

where M is the number of equi-likely transmittable waveforms, λ^2 is the signal-to-noise ratio, and

$$\rho_i = \sqrt{u_i^2 + v_i^2}$$

where the u_i, $i=1,\ldots,M$ and v_i, $i=1,\ldots,M$ from a 2M dimensional zero-mean Gaussian random vector whose covariance matrix is equivalent to the signal correlation matrix. It is hoped that this representation will aid in demonstrating the global optimality of orthogonal signal sets for the non-coherent channel.

The problem reports and publications which were accepted during the last six months include:

References

1. Weber, C. L. and Scholtz, R.A., "Signal Design for Phase-Incoherent Communications", IEEE Trans. on Information Theory, October, 1966.

2. Weber, C. L., "New Solutions to the Signal Design Problem for Coherent Channels", IEEE Transactions on Information Theory, April 1966.

3. Dunbridge, B., "Signal Designs for the Coherent Gaussian Channel", IEEE Trans. on Informations Theory, April 1966.

E. A New Conjecture for the Incoherent Channel

Contract AF-AFOSR-874-66, Air Force Office of Scientific Research

L. R. Welch

The new conjecture implies the Scholtz-Weber conjecture for phase incoherent communications. It is as follows: Let $Z_1, \ldots, Z_n$ be complex normally distributed random variables with $E(Z_i Z_j) = 0$, $E(Z_i \overline{Z}_j) = \Gamma_{ij}$ and $\Gamma_{ii} = 1$, $i=1,\ldots,n$. Let $\rho = \max_i |Z_i|$ and let $F_\Gamma(\rho)$ be the distribution function of ρ. The conjecture is that $F_\Gamma(\rho) \geq F_I(\rho)$ for all ρ where I is the nxn identity matrix.

F. Decoding and Spectral Analysis

Contract AF-AFOSR-874-66, Air Force Office of Scientific Control

L. R. Welch

The theory of group characters provides a unifying treatment of diverse problems. As one example, certain group codes can be regarded as group characters on the additive group of $GF(2^n)$ and the decoding is just

Fourier analysis for that structure. As nother example, ordinary Fourier analysis on N points is just computation of coefficients of group characters on the additive group of integers modulo N. As a third example, deep space ranging code acquisition, is the problem of determining which character best fits the received signal where the characters are again on $GF(2^n)$. An efficient algorithm has been developed with applications in each of these areas. The algorithm work factor is proportional to N log N where N is a) code length, b) number of spectral points, c) ranging code component length.

G. Optimal Channel Signals for Synchronized Multiplexed Communications

Grant AF-AFOSR-496-66, Joint Services Electronics Program

R. M. Gagliardi

In a multiplexed communications system a set of data sources communicates to a corresponding set of individual receivers through a single channel signal. In this research effort we have derived the optimal channel signal that maximizes the average detection probability during each bit period when the data sources are producing carrier modulated binary data in synchronism. The solution is obtained by a fairly straightforward application of Lagrangian multipliers, subject to an average power constraint, but the difficulty is in interpreting the result. In the case of high receiver noise the optimal channel signal approaches the maximum correlation signal; i.e., the signal that crosscorrelates most with the set of data signals to be sent in a given bit period. In the case of low receiver noise, the optimal channel signal is that which crosscorrelates with the data set with the maximum number of positive correlations.

H. Maximum Likelihood Estimation of Signal to Noise Ratio

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

R. M. Gagliardi, C. M. Thomas

In this research effort we have considered the problem of obtaining a maximum likelihood estimate of signal to noise ratio of an ob erved signal. It has been shown that the problem is basically a "noise in noise" type of estimation, when the signal and noise are stochastic, with unknown power levels. Using a stochastic Fourier series expansion of the processes involved, a system interpretation is given for the maximum likelihood estimator. It can be shown that the system corresponds to a "matching" of the estimated observed signal spectrum with the spectra of the signal and noise individually, followed by a division. One may alternatively interpret this system as a bank of spectrum analyzers followed by suitable weighting coefficients whose value depends upon the known spectra of the signal and noise. Similar system interpretations can be made for the forms of expansions of the observed signal. A special case occurs when the signal portion of the observable is not random, but known, except for its amplitude (power level). In this coherent case, involving Gaussian noise, the M. L. E. of signal and noise is identical to the simultaneous M. L. E. of the mean and variance of a Gaussian process.

I. Detection of Periodically Rotating Radar Targets

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

J. L. Wong, I. S. Reed, Z. A. Kaprielian

A theoretical model for the radar echo from a random collection of rotating dipole scatterers has been developed. The basis for the analysis can be described as follows: An r.f. pulse is transmitted from a radar toward a cloud of random dipole scatterers moving about and reflecting energy independently of one another. In addition, each dipole is assumed to have a periodic rotation about an arbitrary axis perpendicular to its length. Since the transmitted wavefront intercepts scatterers at different ranges, the echoes returned to the radar will arrive at a rate which depends upon the local density of the cloud. If the scatterers are statistically independent, the arrival of an echo at time t does not affect the probability of echo arrivals at other times. Thus, the echo arrivals constitute a non-stationary Poisson process. The time-varying correlation function is derived in terms of the characteristics of the transmitted waveform, polarization, and the probability distribution of the scatterers. Under proper assumptions, the time dependence can be eliminated and an expression for the power spectrum of the echo waveform can be derived. A summary of the results of this work is given in USCEE Report #158, entitled "A Model for the Returned Echoes from a Random Collection of Rotating Dipole Scatterers".

The analysis developed can be applied to the study of echoing characteristics of a wide class of scattering clouds and clutter targets. For example, chaff, which is a form of countermeasure used against radar, usually consists of a large number of thin metallic strips (dipoles). When chaff dipoles are dispensed from a moving craft in space, the effects of ejection forces, body instability, and other aerodynamical properties can cause the dipoles to rotate. Rotational motion of the scatterers gives rise of variations of the returned echo power and phase, which in general cannot be neglected.

The foregoing work assumes that both the transmitter and receiver have the same polarization. In radar operations it is often desirable to have different polarizations for transmitting and receiving. The current efforts are to extend the analysis to examine the characteristics of the echo waveform when different transmitter and receiver polarizations are employed.

J. Synthesis of Ambiguity Functions

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

L. K. Montgomery, I. S. Reed

The transform property of ambiguity functions found by Titlebaum [1] has been extended to higher dimensions. Define

$$K_{1234}(\tau, \omega, \rho, \alpha) = \theta_{12}(\tau - \rho, \omega + \alpha)\,\overline{\theta}_{34}(\tau + \rho, \omega - \alpha)$$

where

$$\theta_{mn}(\tau, \omega) = \int_{-\infty}^{\infty} u_m\left(t - \frac{\tau}{2}\right)\overline{u}_n\left(t + \frac{\tau}{2}\right) e^{-j\omega t}\, dt$$

is the signal cross-correlation function of $u_m(t)$ and $u_n(t)$. The four dimensional transform property found is

$$\frac{1}{2\pi}^{2} \iint\iint_{-\infty}^{\infty} d\tau_1\, d\tau_2 \Big\{ K_{1234}(\tau_1 - \delta_1, \omega_1 + \lambda_1, \tau_2 + \delta_2, \omega_2 - \lambda_2)$$

$$\cdot \overline{K}_{5678}(\tau_1 + \delta_1, \omega_1 - \lambda_1, \tau_2 - \delta_2, \omega_2 + \lambda_2) \exp\Big[j(\omega_1 x_1 - \tau_1 y_1 - \omega_2 x_2 + \tau_2 y_2) \Big]$$

$$= \frac{1}{4} K_{6237}\left(\frac{1}{2}x_1 - \delta_1, \frac{1}{2}y_1 + \lambda_1, \frac{1}{2}x_2 + \delta_2, \frac{1}{2}y_2 - \lambda_2\right)$$

$$\cdot \overline{K}_{5148}\left(\frac{1}{2}x_1 + \delta_1, \frac{1}{2}y_1 - \lambda_1, \frac{1}{2}x_2 - \delta_2, \frac{1}{2}y_2 + \lambda_2\right)$$

By substituting particular values of the parameters, many new transform properties of ambiguity and signal correlation functions are obtained. The transform properties in higher dimensions are similar to the above formula.

By using the signal cross-correlation function as an expansion kernal, it was found that any arbitrary signal F(t) can be expressed as a double integral over chosen elementary signals. This generalizes Helstorm's result[2] in which only Gaussian elementary signals were used. The elementary signals are defined as

$$\psi(t;\tau,\omega) = \bar{G}(t-\tau)\exp\left[j\omega t - j\frac{\omega\tau}{2}\right]$$

where G(t) is a signal normalized to have unit energy. If the expansion kernal $g(\tau,\omega)$ is chosen as the signal cross correlation function of G(t) and $\bar{F}(t)$ then

$$F(t) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} g(\tau,\omega)\psi(t;\tau,\omega)d\tau\, d\omega \ .$$

By Titlebaum's transformation[1]

$$\frac{1}{2\pi}\iint_{-\infty}^{\infty} \theta_{FF}(\tau-\delta, \lambda+\omega)\bar{\theta}_{GG}(\tau+\delta, \lambda-\omega)\exp\left[j(\lambda x-\delta y)\right]d\delta\, d\lambda$$

$$= g(x-\tau, y+\omega)\,\bar{g}\,(x+\tau, y-\omega) \ .$$

Substituting $\tau = -x$ and $\omega = y$ determines $g(\tau,\omega)$, and consequently F(t), to within a multiplicative constant, so this gives a new method of synthesizing a signal correlation function.

The study of using phase modulated signals to produce an ambiguity function having the shape of an inverted thumbtack led to the necessity of evaluating the integral

$$I(t,\alpha) = \int_{-\infty}^{\infty} \exp\lfloor j(\omega^{\alpha} + t\omega)\rfloor \, d\omega$$

which is a generalization of Airy's integral. Writing $\alpha = 2m + \beta$, it was found that $I(t, 2m+\beta)$ converges only for $m = 1, 2, \ldots$ and $0 \leq \beta \leq 1$. In general $I(t,\alpha)$ is expressed as an infinite series, but for α an integer, the integral can be expressed as a finite sum of generalized hypergeometric functions.

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K. Optical Coherence Theory

Grant AF-AFOSR-188-66, Air Force Office of Scientific Research

R. S. Macmillan

Considerable progress in optical coherence has been made during the past six months. The theoretical foundations for the analysis and optimization of optical system by means of maximization of the trans-coherence function have been formulated and substantiated. The optimization procedure has been applied to linear space-time problems, yielding results which are physically reasonable and which agree with known results in special cases.

A problem which has received attention recently is the effect of a random medium on coherence and the corrections which must be made by a lens system to compensate for these effects.

The coherence effort has generated several auxiliary studies in related subjects. A paper has been written on a new method of finding probability distributions after memoryless transformations. The method yields known results with less labor and is particularly applicable to computer solutions of problems. Another related paper has been written on optical polarization modulation.

L. Speech Signal Processing

Contract Nonr-228(34), Office of Naval Research

S. W. Golomb, R.A. Scholtz, L. R. Welch, W. J. Hurd

The amplitude moments of filtered and unfiltered speech and interference are being investigated in order to find methods of distinguishing speech from other sounds. Results indicate that speech can be distinguished from Gaussian noise by measuring the ratio of the fourth moment to the square of the second moment, where the moments are measured over one second intervals. This result appears to hold for both male and female speakers, and for all the languages tested. Experiments are planned to determine the length of time required to detect the presence of speech as a function of speech-to-noise ratio and type of noise.

M. Communication Theory Research

Contract 951076, Jet Propulsion Laboratory

S. W. Golomb

The moment generating function of classical probability theory is both a convenient means of calculating the moments of a distribution, and an effective embodiment of the properties of the distribution for various analytical purposes. In this correspondence, a generating function is defined whose derivatives, evaluated at a certain place, yield the moments of the self-information of the distribution. The first of these moments is the now-familiar "entropy" of the distribution. As a formalism, this technique works equally well for discrete and continuous distributions. Moreover, the "information-generating function" summarizes those aspects of the distribution which are invariant under measure-preserving rearrangements of the probability space.

Results in a recent paper[1] indicate that given a probability distribution we can compute the corresponding "information generating function" $T(u)$, from which the entropy of the distribution, and other relevant parameters, are readily extracted. Examples have been worked out for several of the most familiar discrete and continuous distributions. The significance of the convergence region for $T(u)$ has been discussed, and the problem of reconstructing the distribution given the information generating function has also been considered.

The literature in statistical communication theory generally contains a significant shift in viewpoint between the discrete and the continuous case. In the latter context, a particular distribution is assumed almost from the outset, and most of the theorems refer to such things as the "white gaussian noisy channel", or other equally specific assumptions. For the discrete case, on the other hand, the results are rarely evaluated in terms of specific distributions. A paper[2] presents remarks that are intended as a step in this direction, viz. the explicit form which Huffman coding assumes when applied to the geometric distribution. It would also be appropriate to have explicit answers for the binomial distribution, the Poisson distribution, etc.

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N. Automata and Game Theory

Grant AF-AFOSR-874-66, Air Force Office of Scientific Research

S. W. Golomb

Results[1,2] relating to computability and decidability of certain classes of mathematical games and puzzles have been produced. In a paper[1], we formulate a vector space model of all such games of "take-away", examining the set-theoretic relations between the set of permitted moves and the set of winning (or losing, or drawing) positions. In the one-dimensional case we discuss several of the more interesting games in detail, and arrive at a recursion relation, in general, for classifying winning vs. losing positions. This recursion can be "mechanized" with a simple nonlinear shift register. Finally, we define a much more general class, called "progressive games", to which a more complex recursive analysis is applicable.

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3.3 CODING THEORY AND AUTOMATA STUDIES

A. Codes with Synchronization Capability

Grant AF-AFOSR-874-66, Air Force Office of Scientific Research

R. A. Scholtz

Research during the past six months has been confined primarily to developing variations of the construction procedure for synchronizable codes reported in the previous progress report. The major product of this research is a simple algorithm for constructing maximal comma-free codes of odd word length. The algorithm involves an iterative construction procedure in which a synchronizable code C_n is constructed from another synchronizable code C_{n-1} by removing a word from C_{n-1} and using it as either a prefix or suffix of other words from C_{n-1} depending whether it has even or odd length. This algorithm has been successful in all cases tested but has not been proven generally applicable.

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B. Coding the Vertices of a Graph

Grant AF-AFOSR-496-66, Joint Services Electronics Program

M. A. Breuer

Research has continued on the problem of coding the vertices of a graph. The problem is the following. Given a graph G of n nodes. Assign to each node i a unique binary code c_i of length m such that the Hamming distance between c_i and c_j is less than or equal to T if and only if nodes i and j are connected. A third proof of the existence of these codes has been developed based upon a pure synthesis procedure. The following rather surprising result was obtained: {G is T doable} {G is T+1 doable}. Applications of the coding technique to information transmission and the state assignment problem have been found.

In order to obtain optimal codes for this problem (i.e. minimal T or minimal m) the synthesis problem is being programmed using the backtrack programming concept.

Part of this work will appear in the June issue of the IEEE Trans. on Information Theory, and was presented at the 1966 IEEE International Conf. on Information Theory.

C. Some State Assignment Problems for Asynchronous Switching Circuits

Grant AF-AFOSR-1018-66, Air Force Office of Scientific Research

G. A. Bekey, D. C. Collins, W. S. Meisel

Most work to date has been devoted to solving the problem of minimizing the number of secondary variables necessary to code a machine. The constraint adopted for this problem was that all transitions take place in one or more steps of one variable change at a time, i.e. cycling coding.

An efficient tabular reorganization of information in merged flow tables, which is pertinent to the state assingment problem, has been developed. Using this device three algorithms have been developed for assigning the coding for any given flow table. These algorithms do not, as yet, insure a code with a minimum number of secondary variables; but two of the algorithms provide rapid hand methods for cut and try, while the third can be used for large tables when implemented on a computer. All three of these algorithms yield either a minimum, or close to minimum, coding.

The problem of developing completely general coding assignments which will work for any table has been investigated. Assignments have been found which seem to satisfy any flow table of eight rows (or less) and sixteen rows (or less). Both of these assignments require the use of fewer secondary variables than Huffman's bound of $2S_0 - 1$.

Future work will be devoted to abstracting properties of these general assignments to enable generation of codes which will handle flow table 64, 128, 256 row length. A nonexhaustive proof of the complete generality of these assignments will be developed. In addition the problem of finding algorithms which can be implemented on a computer and which will yield an absolute minimum number of secondary variables for any given flow table will be investigated.

D. Hazards in Combinatorial and Sequential Switching

Grant AF-AFOSR-1018-66, Air Force Office of Scientific Research

W. S. Meisel, D.C. Collins

Hazards in switching circuits result in unpredicted and incorrect operation of those circuits as a result of delays. The study of hazards is an attempt to analyze such behavior and develop methods to prevent it.

A survey of the literature in this field was completed. A paper[1] which attempts to: 1) unify and clarify the existing nomenclature, 2) report original results on the derivation of meaningful and useful solutions to the problems of identifying and correcting hazards, and 3) suggest significant unsolved problems.

An examination of hazards and their significance to coding techniques was begun. The effect upon hazard-free circuit operation of the use of non-critical races in an asynchronous sequential circuit. It is shown[2] that the removal of single-input-change static hazards in the networks realizing the next-state functions is not, in general, sufficient to prevent static hazards if a non-critical race is present. A necessary and sufficient condition for hazard-free realization of a flow table containing non-critical races is given, and it is shown that a hazard-free realization is always possible.

Future work will include the following efforts: (1) graphical techniques for the design of hazard-free, multi-level combinational circuits and (2) the analysis of multiple-order hazards with first-order hazards present.

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3.4 HYBRID AND DIGITAL COMPUTATION

A. Re-entry Tracking of Ballistic Vehicles by Hybrid Computation

Contract AF 04(695)-746, Space Systems Division, Air Force Systems Command

R. B. McGhee

An iterative parameter estimation scheme based on nonlinear least squares regression has been successfully programmed and tested as part of this research activity. The method used was essentially that proposed by Bekey and McGhee[1] although the partial derivatives of the re-entry trajectory with respect to the various re-entry parameters were obtained by finite difference methods rather than by solving sensitivity differential equations. No convergence problems were introduced by this approximation. The use of finite differences results in at least a two to one reduction in the amount of analog computer equipment required and, in addition, permits the same basic re-entry simulation to be used either for regression or Bayesian estimation[2].

In comparison to the previous purely digital solution of the re-entry tracking problem[3], the hybrid solution requires roughly one-tenth as much computing time. This means that, under the conditions assumed, a digital computer capable of tracking one target in real time can handle approximately ten targets simultaneously if it is furnished with a high speed analog computer (or digital differential analyzer) for solving the re-entry differential equations.

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B. Identification of Nonlinear Dynamic Systems by Hybrid Analog-Digital Computation

Grant GP-3113, National Science Foundation

R. B. McGhee, R. B. Walford

Work on this problem continues to be concentrated in the area of optimum sampling for Monte-Carlo integration[1]. By making use of fitted Gaussian distributions, it has been possible to obtain a sampling scheme which minimizes the generalized variance of the parameter estimates. A report describing these results is in preparation.

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C. Design Automation of Computers

Grant AF-AFOSR-496-66, Joint Services Electronics Program

M. A. Breuer

Work was completed on formulating a component placement problem and a digital computer backboard wiring problem as integer linear programs. The component placement problem consists of making a unique assignment of components to column positions such that wireability is maximized. The backboard wiring problem consists of three inter-related sub-problems, namely, the placement, the connection, and the routing problems. The placement and connection problems are combined and solved as one, thereby giving the optimal circuit connections as well as minimizing the total lead length. It is shown that under certain assumptions, the number of inequalities and variables in the problem can be greatly reduced. Further simplifying assumptions lead to near optimal solutions.

The following concepts were formulated as linear inequalities:

1) The absolute magnitude of the difference between two variables.
2) Minimizing the minimum function of a set of functions.
3) Counting the number of (0,1) adjacent component pairs in a vector.

The results of this work have been accepted for publication in the Naval Research Logistics Quarterly.

In the area of Boolean switching theory minimization, the problem of simplifying the Quine matrix has been under consideration. A concept called Proper Reduction was developed a few years ago. Presently, the statistical properties of this reduction technique are being determined. Also, a computer program is being written so that experimental results can be obtained.

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